Int. J. Heat Mass Transfer. Vol. 6, pp. 53–78. Pergamon Press 1963. Printed in Great Britain.

NUCLEATE BOILING. THE REGION OF ISOLATED BUBBLES AND THE SIMILARITY WITH NATURAL CONVECTION

NOVAK ZUBER

General Engineering Laboratory, General Electric Company, Schenectady, New York

(*Received* 2 *July* 1962)

Abstract—Experimental data indicate that nucleate boiling consists of two regions, (a) the region of isolated bubbles and (b) the region of interference. The vapor removal pattern, the flow pattern and the mechanisms of heat transfer in the two regions are discussed and analysed. A criterion for the change from one region to another is presented.

In the regime of isolated bubbles, bubbles do not interfere with each other and at any particular point vapor is produced intermittently. Jakob's description of the natural flow circulation in nucleate pool boiling from a horizontal surface is similar to Malkus' and Townsend's description of the flow regime in turbulent natural convection from a horizontal surface. In both cases the heat transfer is caused by the "up-draught" induced circulation. It is shown that the same equations which predict the heat-transfer coefficient and the average turbulent velocity fluctuation in natural turbulent convection from a horizontal surface can be used in the regime of isolated bubbles if the vapor void coefficient i.e. the vapor hold-up is taken into account in evaluating the mean density of the fluid. Equations relating the vapor void coefficient to the heat-transfer coefficient or to the bubble population density and liquid superheat temperature are presented. It is shown that an upper limit exists for the heat-transfer mechanism induced by the "up-draught" circulation. Equations predicting the limiting value of the heat-transfer coefficient and of the heat flux density in the regime of isolated bubbles are presented also. All these results, predicted by the analysis, are shown to be in qualitative and quantitative agreement with experimental data presently available.

In the region of interference, bubbles interfere with each other to form continuous vapor columns and patches. Vapor is continuously produced by vaporization of a pulsating micro-layer (proposed and described by Moore and Mesler) at the base of a vapor column or of a vapor patch. In this regime the dominant heat-transfer mechanism is, most probably, the latent heat transport process (formulated by Gaertner) and the latent heat transport associated with the large bursts of vapor caused by collapsing vapor patches.

The results of the analysis indicate that a particular regime of nucleate pool boiling (a two-phase problem) can be analysed as a turbulent natural convection problem (a single-phase problem). Applications of similar considerations to other aspects of the two-phase flow appear therefore promising. The important effect of the two-phase flow patterns on the mechanism of heat transfer and on the coefficient of heat transfer for a two-phase mixture is demonstrated again, emphasized and discussed.

NOMENCLATURE

(Dimensions in the $ML\theta T$ System)

a,	thermal diffusivity $[L^2\theta^{-1}]$;
A,	total heat transfer area $[L^2]$;
B,	dimensionless group defined by equation (20);
c,	specific heat of the liquid at constant pressure $[HM^{-1}T^{-1}]$;
d,	characteristic dimension $[L]$;
d_c,	diameter of a cavity on the heating surface $[L]$;
D,	diameter of a bubble $[L]$;
D_b,	diameter of a bubble departing from a horizontal heated surface, diameter i.e. of a "break-off" bubble $[L]$;
D_{bF},	diameter of a bubble defined by equation (5) $[L]$;
f,	frequency of bubble emission $[\theta^{-1}]$;
g,	acceleration due to gravity $[L\theta^{-2}]$;
N_{Gr},	Grashof modulus;
h,	heat transfer coefficient $[HL^{-2}\theta^{-1}T^{-1}]$;

h_{fg}, latent heat of vaporization $[HM^{-1}]$;
k, thermal conductivity of the liquid $[HL^{-1}\theta^{-1}T^{-1}]$;
n, number of bubbles;
n/A, number of bubbles per unit surface $[L^{-2}]$;
N_{Nu}, Nusselt modulus;
N_{Pr}, Prandtl modulus;
Q_v, volumetric flow of the vapor $[L^3\theta^{-1}]$;
$\dot{Q}/A$, heat flux density $[HL^{-2}\theta^{-1}]$;
s, center to center spacing of bubbles, defined by equation (26) $[L]$;
T, absolute temperature $[T]$;
T_s, absolute saturation temperature $[T]$;
T_w, absolute temperature of the solid $[T]$;
$T = T_w - T_s$, liquid superheat temperature difference $[T]$;
t_d, delay time $[\theta]$;
t_b, bubble break-off time $[\theta]$;
U_t, terminal velocity of bubble rise $[L\theta^{-1}]$;
U_v, velocity of bubble rise defined by equation (14) $[L\theta^{-1}]$;
U_{sv}, superficial velocity of the vapor, defined by equation (11) $[L\theta^{-1}]$;
$\bar{v}_L$, average turbulent velocity fluctuation $[L\theta^{-1}]$;
α, volumetric vapor fraction, i.e. vapor hold-up, i.e. vapor void coefficient;
β, coefficient of thermal expansion $[T^{-1}]$;
ρ, mass density $[ML^{-3}]$;
$\Delta\rho = \rho_L - \rho_v$, density difference $[ML^{-3}]$;
σ, surface tension $[M\theta^{-2}]$;
ν, kinematic viscosity of the liquid $[L^2\theta^{-1}]$;
θ, contact angle in equation (11) in degrees;
τ, period of bubble emission $[\theta]$.

Subscripts

v, vapor;
L, liquid;
m, mixture;
w, evaluated at the wall;
∞, bulk.

1. INTRODUCTION

1.1. *Previous analyses and correlations*

THIS paper considers the problems of heat transfer from a horizontal surface to a wetting liquid in the nucleate boiling regime. More than two dozen equations have been heretofore proposed for correlating experimental data; some of the more commonly used ones are discussed and evaluated in [1]. Because the high heat flux densities in nucleate boiling were attributed to bubbles which induce locally a strong agitation of the liquid near the heating surface most of the correlations have been formulated in terms of a bubble Reynolds number and of a bubble Nusselt number. All of them can be put in the form

$$h = \text{const.}\,(T_w - T_s)^m \tag{1}$$

where the value of the exponent varies between 1 and 3 and the constant depends on the thermodynamic properties of the vapor and the liquid as well as on the solid–liquid combination. It was thought, originally, that the effect of surface condition, of the contact angle, etc. can be taken into account by a suitable adjustment of the multiplying constant.

Recent experimental results indicate, however, that the proposed correlations are not general and that an equation of the form (1) may not be adequate to describe the phenomenon. It was noted and discussed [2, 3, 4, 5, 6, 7, 8] that a generalized correlation cannot be expected unless the correlation takes into account the nucleating characteristic of the heating surface and the effect of bubble population. Experiments reported in the literature [5, 9, 10] indicate that different nucleating characteristics of the surface, and the bubble population, both affect not only the value of the multiplying constant in (1) but also the value of the exponent. For example, variations in the exponent m, ranging from 1 to 25 can be produced by polishing the surface with different grades of emery paper [5].

In addition to providing data on surface effects, recent experiments provided also quantitative information on the bubble population density. It was shown [11, 12, 13, 14] that instead of expressing the heat-transfer coefficient in terms of the liquid superheat temperature difference $(T_w - T_s)$ as in (1), it was also possible to express it in terms of the bubble population density alone thus

$$h = \text{const.} \left(\frac{n}{A}\right)^a \tag{2}$$

where the value of the exponent a was $a = 1/3$ for the data in [11, 12, 13] and $a = 0{\cdot}42$ for the data in [14].

In conclusion, recent and more detailed experimental information has conclusively shown that the heat flux density in nucleate boiling is not a single-valued function of the superheat temperature difference, but depends upon both the superheat and the bubble population. Indeed, it was first shown experimentally by Yamagata and Nishikawa [11, 12] (in experiments performed by varying the surface tension) that an expression

$$\frac{\dot{Q}}{A} = \text{const.}\ (T_w - T_s)^b \left(\frac{n}{A}\right)^c \tag{3}$$

approximates well the data. The value of the exponents thus found were: $b = 3/2$ and $c = 1/4$. Two-parameter expressions, similar to (3), were also recently derived from boundary-layer considerations [8, 15]. The value of the exponents were however slightly different, with $b = 2$ and $c = 1/4$ obtained in [8] and $b = 1$ and $c = 1/2$ obtained in [15].

1.2. *Purpose and outline of the paper*

It is the aim of this paper to examine both the fluid dynamic and the heat transfer processes in nucleate pool boiling from a horizontal surface.

The fluid dynamic problem is analysed by considering first the flow regimes induced by a single bubble (Section 2.1). In Sections 2.2 and 2.3 these results are used to analyse, quantitatively, the flow regimes induced by a swarm of bubbles. The analysis substantiates Jakob's [16, 17] description of the flow in nucleate boiling. It is concluded that this flow is similar to that described by the theories of Malkus [18, 19] and of Thomas and Townsend [20, 21] for turbulent natural convection from a horizontal surface.

The heat-transfer problem is formulated then by considering the similarity between nucleate boiling and natural convection, a similarity which was noted and discussed already in [11, 12]. It is shown in Sections 3.1 and 3.2 that the equation which predicts the heat-transfer coefficient in natural convection from a horizontal surface can be used for predicting heat-transfer rates in nucleate pool boiling if the density of the medium is modified to take into account the volumetric vapor fraction (i.e. the vapor hold-up or the vapor void coefficient) at the heating surface. When the thermal expansion of the liquid is neglected a two-parameter equation of the form of (3) results. It is shown also in Sections 3.1 and 3.2 that when the volumetric vapor fraction is taken into account the turbulent velocity fluctuations in nucleate boiling can be estimated from the theory of Malkus.

A discussion of the results is given in Part 4. Section 4.1 considers the question of hydrodynamic similarity. The region of isolated bubbles and the effect of bubble interference is discussed in Section 4.2. The maximum values of the heat-transfer coefficient and of the heat flux density in the region of isolated bubbles are evaluated in Sections 4.3 and 4.4 respectively. The effect of the change in the two-phase flow regimes on the mechanism of heat transfer in nucleate pool boiling is considered in Section 4.5.

1.3. *Significance of the results*

The results of the analysis indicate that nucleate pool boiling consists of two regimes (a) the region of isolated bubbles and (b) the region of interference.

Equations which predict (1) the heat-transfer coefficient, (2) the average turbulent velocity fluctuation, (3) the vapor volumetric fraction (vapor hold-up) as well as the limiting values of (4) the bubble spacing, (5) the bubble population density, (6) the heat-transfer coefficient and (7) the heat flux density in the region of isolated bubbles are presented and satisfactory agreement of predicted values with experimental data is shown.

The analysis indicates that a particular regime of nucleate pool boiling (a two-phase flow problem) can be analysed as a turbulent natural convection problem (a single-phase flow problem).

The heat-transfer mechanism described in this paper is due to the bubble stirring action and bubble-induced flows in the boundary layer adjacent to the heating surface. In this sense, the present analysis can be looked upon as further elaboration of the bubble agitation mechanism first proposed by Jakob [22, 23] and explored and amplified further by Rohsenow [24], Rohsenow and Clark [25], Gunther and Kreith [26] and by Forster and Zuber [27] in this country, and by Kutateladze [28] and Kruzhilin [29, 30] in Russia.

However, recent experiments indicate that as the nucleate heat flux is increased, the transfer problem changes from one caused by the bubble stirring action to one which is a transport of latent heat due to the evaporation at the base of the vapor column (experiments of Moore and Mesler [31] for liquids at saturation temperature), or due to the evaporation at the bubble base (experiments of Bankoff and Mason [32] for sub-cooled liquids).

The occurrence of a change in the nature of the transfer process as heat flux is increased in nucleate pool boiling of liquids at saturation temperature was first advanced by Zuber [8] from an analysis of the Gaertner–Westwater data [14]. It was observed that this change takes place when the average bubble spacing becomes less than two bubble diameters. This observation was also confirmed later by Hsu and Graham [33] from shadowgraph observation of the boundary layer.

The analysis presented in this paper confirms the observation that as the heat flux increases from incipient boiling to the critical heat flux (often referred to as the "burnout" heat flux) the process of heat transfer changes from one effected by the bubble agitation to one due to the transport of latent energy, the latter process being also in agreement with the hydrodynamic instability theory for the critical heat flux [34, 35, 7, 36, 37].

1.4. *The extension of the method to an analysis of boiling heat transfer in forced convection*

Some of the ideas discussed and presented in this paper can be modified and extended to consider heat-transfer rates to liquids in forced convection boiling inside ducts. However, such an extension (which will be discussed in another part) can be applied only to certain flow patterns of the two-phase mixture. Equations which have been obtained by correlating experimental data for pool boiling from horizontal surfaces or cylinders cannot and should not be used alone for predicting heat-transfer rates to boiling liquids flowing inside pipes.

It was shown in a number of recent publications, [1] and [2] among others, that a considerable error can be made in evaluating the heat-transfer coefficient if the effects of convection (forced or buoyant) and of the flow patterns are neglected. Statements to the contrary which have been made previously in the literature are of limited validity and can only mislead the unwary designer and reader.

2. THE FLUID FLOW PROBLEM

As in any other convective process the heat transfer to a boiling liquid depends upon the flow conditions of the fluid. An understanding of the flow and of the flow regimes is, therefore, a prerequisite for an analysis of the process of heat transfer. In nucleate boiling this entails an understanding of the processes associated with the vapor generation (nucleation and bubble growth), and with the problem of vapor removal. In Section 2.1 we analyse first the flow induced by the generation and the removal of a single bubble. In Sections 2.2 and 2.3 these results are used to analyse the flow induced by a swarm of bubbles.

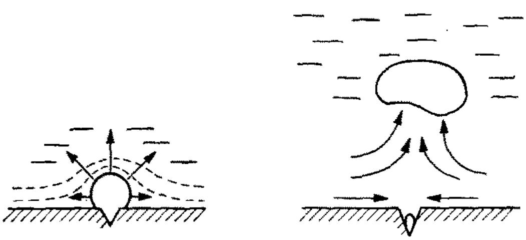
2.1. *The flow regimes induced by single bubbles*

2.1.1. *The bubble growth—the source flow.* Following the nucleation from a cavity, the bubble grows in a superheated liquid film adjacent to the heating surface. It was shown in [8] that the bubble growth predicted from the energy considerations of Bošnjaković and Jakob i.e. from the equation proposed by Fritz and Ende [38]

$$D = 4\frac{(T_w - T_s)c\rho_L}{\rho_v h_{fg}}\sqrt{\left(\frac{at}{\pi}\right)} \tag{4}$$

approximates satisfactorily the experimental data of Staniscewski [39] for water and methanol in pool boiling at saturation temperature. More recently Strenge, Orell and Westwater [40] reported that an equation of the form (4) can be used to approximate the bubble growth rates in pool boiling of ether and pentane. It is important to note again [8] that local conditions in pool boiling are neither known nor measured. Consequently, (4) with the average value of $(T_w - T_s)$ can predict only the average growth but not the growth of a specific bubble (unless it coincides with the average one).

Although the bubble slightly deforms (it flattens) while growing attached to the surface, or sufficiently small bubbles the liquid flow associated with the bubble growth period can be analysed as a source flow (Fig. 1a). A mean velocity in the liquid associated with this kind of flow is given in Appendix A. Conceptual models based on the source flow were formulated by Bankoff [41, 42] for nucleate boiling of sub-cooled liquids and by Zuber [8] for liquids at saturation.



(a) (b)

FIG. 1. A schematic representation of the source flow and of the wake flow associated with the growing and departing bubble.

2.1.2. *The departure of a bubble—the wake flow.* A bubble grows and remains attached to the surface until, at time t_b, it reaches a characteristic diameter D_b, breaks-off and departs from the heating surface. The departure is governed by the dynamics of the surrounding liquid as well as by the buoyant and adhesion forces. Fritz [43] considered only the static equilibrium between buoyant and adhesive forces, and derived the following expression.*

$$D_{bF} = 0{\cdot}021\ \theta \left[\frac{\sigma}{g(\rho_L - \rho_v)}\right]^{1/2} \tag{5}$$

* We are using the subscript b, to denote break-off and F to denote that it is predicted by the equation of Fritz.

where the contact angle θ, is measured in degrees. Experiments performed with water, CCl_4 and other liquids [16, 39, 14] show that D_b is given by a statistical distribution. The values predicted by (5) were found [16, 8] to be in agreement with the mean value of this distribution.

Immediately after the detachment the lower surface of the bubble re-enters and deforms the bubble in a lenticular shape. Liquid is entrained in the wake of the detaching and rising spheroidal bubble. Consequently, the flow associated with bubble departure can be approximated as a wake flow (Fig. 1b).

The velocity of rise of a spheroidal bubble is given by

$$U_t = \text{const.} \left[\frac{\sigma g(\rho_L - \rho_v)}{\rho_L{}^2}\right]^{1/4} \tag{6}$$

where the value of the constant is 1·18 according to [44] or 1·53 according to [45]. Other expressions for U_t as function of the bubble radius are available in the literature [46].

Following the departure of a bubble, colder liquid comes in contact with the solid and gets heated during a "delay time", t_d, at the end of which time another bubble is nucleated from the same cavity. This new bubble grows until, at time t_b, it in turn departs from the surface and the process is repeated. A bubble column is thus formed by bubbles successively rising from a nucleating center. The duration of the delay time t_d depends upon conditions in the vicinity of the nucleating cavity, i.e. upon the local heating rate, thermal fluctuations in the liquid, and the radius of the cavity. The duration of the "break-off time", t_b, depends upon the local superheat temperature difference and on the local hydrodynamic conditions. For a given cavity, both t_d and t_b vary from run to run [39, 40] and vary for different cavities as well. Consequently the frequency of bubble emission

$$f = \frac{1}{\tau} = \frac{1}{t_b + t_d} \tag{7}$$

is also given by a statistical distribution [16].

Although both D_b and f are given by statistical distributions, Jakob [23] found that their product remains approximately constant, i.e.

$$D_b \,.\, f = \text{const.} \tag{8}$$

For both water and CCl_4 the value of the constant was found to be 7·7 cm/s. It was reported in [11] that (7) was also in agreement with their experimental data. It is important to note that experiments reported in [23, 16] and in [11] were performed at atmospheric pressure and at relatively low heat flux densities (from about 8000 kcal/h m² to 30 000 kcal/h m²). It is shown in [36] and in Appendix B that for the experimental conditions of [23, 16] and [11], the product $D_b . f$ is given by

$$D_b . f = \frac{1 \cdot 18}{2} \left[\frac{\sigma g(\rho_L - \rho_v)}{\rho_L^2} \right]^{1/4}. \tag{9}$$

On Fig. 2 the values of $D_b . f$ predicted by (9) are compared with experimental data for water [38, 11], for CCl_4 [16], and for methanol [47, 48]; the agreement appears satisfactory.

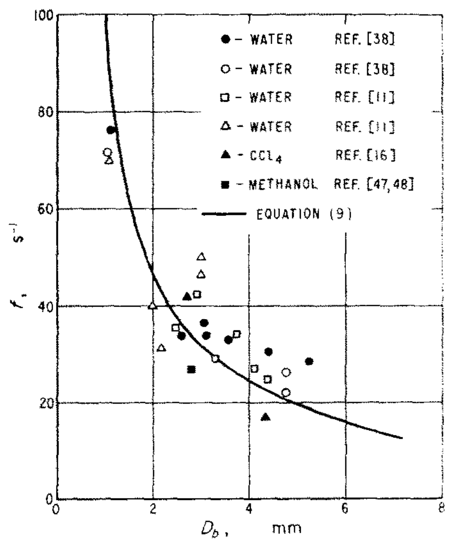


FIG. 2. The relation between the frequency of bubble emission and the diameter of a bubble departing from a horizontal surface.

However, as the heat flux is increased the rates of bubble formation and of bubble growth increase causing bubbles to interfere with each other. Because these interactions change the regimes of bubble removal and, consequently, affect the flow conditions adjacent to the heating surface, they must be taken into account.

2.1.3. *The regimes of bubble removal.* The regimes of bubble removal in nucleate boiling from a horizontal surface were investigated experimentally by Yamagata and Nishikawa [11, 12]. It was noted and discussed by Zuber [36] that the removal of bubbles and the flow regimes described by Yamagata and Nishikawa are identical with those reported by Davidson and Amick [49] for the formation of gas bubbles at orifices. Consequently, the considerable literature on the latter subject [49, 50, 51, 52, 53, 54] can be used to help analyse the regimes of bubble removal in nucleate boiling [10, 11, 47, 48, 55, 56]. This was done in [36] and [57]; three regimes of gas (vapor) removal can be distinguished.

At very low gas (vapor) flow rates the bubble formation is a problem of hydrostatics. The diameter of a bubble can be determined by considering the balance of buoyant and adhesion forces at the orifice (at the nucleating center), thus approximately

$$D = \left[\frac{6\sigma d_e}{g(\rho_L - \rho_v)} \right]^{1/3}. \tag{10}$$

Smaller bubbles are spherical, the larger ones are spheroidal or bell type [11, 12] (Figs. 3a, b). Bubbles rise at a constant velocity without interacting with each other, they are separated and isolated from each other. The bubble volume is nearly independent of the gas (vapor) flow rate, but the frequency of bubble emission increases with increasing flow rate. In the literature this flow regime is referred to as "laminar" or as the region of static, separated or isolated bubbles. The experimental results described in the preceding section pertain to this regime.

At intermediate gas (vapor) flow rates (above a "critical" flow rate) the frequency of bubble formation remains essentially constant while the bubble volume increases with flow rate. The spacing between rising bubbles decreases so that a bubble interacts with its predecessor above the orifice (nucleating center). Coalescence takes place at the orifice. Bubbles are of non-uniform size and have been described as "mushroom like" [43] or "precession" and "tandem" [11, 12] (Figs. 3c, d). In the literature this regime is referred to as "turbulent" or the region of "multiple bubbles".

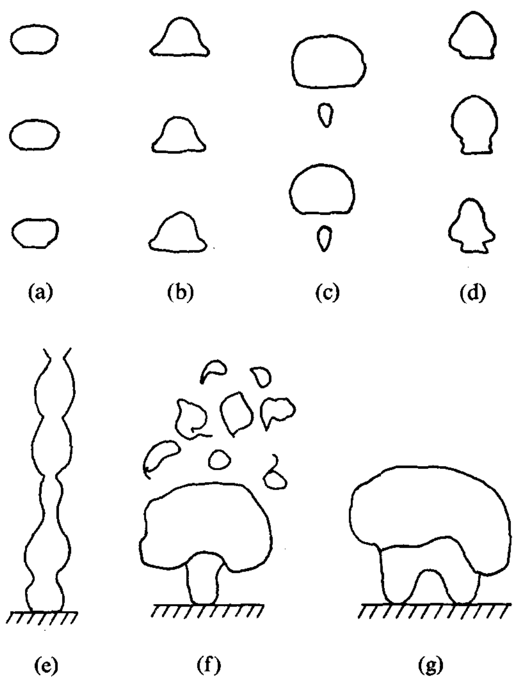


FIG. 3. A schematic representation of the various bubble shapes in nucleate boiling. (Figs. 3a, e are reproduced from the paper by Yamagata and Nishikawa [11]).

At still higher gas (vapor) flow rates a swirling air (vapor) stream is generated at the orifice (nucleating center). The jet of gas was found to be similar to a tornado [53, 58] or a waterspout [53]. Large, irregular, swirling bubbles in the discontinuous jet are shattered into small bubbles 3–4 in above the orifice (Figs. 3e, f). Swirling, continuous vapor columns in nucleate boiling are reported in [10] and [11].

In nucleate boiling a bubble can interact not only with its predecessor rising above the nucleating site but it can interact also with neighboring bubbles on the surface. Similarly two or more swirling continuous vapor columns can interact with each other [11]. An interaction between two nucleating sites is shown on Figs. 3g, 4.* This photograph indicates that the vapor patch grows while attached to the surface by the two original stems; the evaporation occurs probably at the base of these stems.

The considerations in the preceding sections lead to the conclusion that the flow regimes of vapor removal from a single nucleating center change with increasing rates of vapor generation. At low vapor flow rates the removal is in the form of isolated bubbles whereas at high flow rates it is in the form of continuous swirling vapor columns and vapor patches. The liquid flow in the regime of isolated bubbles can be idealized as a source flow during the bubble growth and as a wake flow after the bubble departure from the heating surface.

2.2. *The flow regimes induced by a swarm of bubbles*

In order to analyse the flow and the flow regimes induced by a swarm of rising bubbles in nucleate pool boiling it is advantageous to consider the similarity between nucleate pool boiling from a horizontal surface and the process of gas bubbling through a porous plate. This similarity was analysed by Zuber [36] and more recently by Wallis [59]; it was shown that both the requirement for initiating the bubbling process and the flow regimes in these two phenomena are similar. Consequently, the information which is available in the literature [50, 52, 59, 60, 61, 62, 63] on the hydrodynamic conditions during the process of bubbling from a porous or perforated plate can be used to analyse the process of bubbling in nucleate pool boiling.

From experiments performed with air bubbling from perforated plates and porous surface Siemes [51] concluded that as long as the spacing between the active bubbling centers (pores or perforations) was greater than the diameters of bubbles at departure then the regimes of bubble removal were similar to those observed with single orifices. The experimental investigations [50, 52, 59, 60, 61, 62, 63] indicate that the process of gas bubbling from porous or perforated plates is characterized by three distinct flow regimes referred in the literature as "laminar", "turbulent" and oscillating "slug" or "plug" flow.

The "laminar" flow regime exists at low gas

* This photograph was taken by Mr. R. Semmeria from the Commissariat à l'Énergie Atomique, Centre d'Études Nucleaires de Grenoble, France. The author is indebted to Mr. R. Semmeria for giving him this photograph and for his permission to reproduce it herein.

flow rates. Bubbles of constant volume rise without interacting with each other. This regime corresponds to the "laminar" regime in bubbling from an orifice. In the "laminar" regime the liquid ahead and behind rising bubbles is at rest; no gross liquid circulation exists in the field. At these low flow rates an increase of gas flow results mostly in an increase of the number of active pores, i.e. of the bubble population.

The "turbulent" regime is characterized by large liquid convection currents induced by rising bubbles. It can exist at low gas (vapor) flow rates if the liquid is set in motion by displacement and entrainment in the wakes of rising bubbles. It always exists at higher gas flow rates. In the "turbulent" flow regime an increase of gas flow rates results in increasing both the bubble population and the bubble volumes. Bubbles in this regime are of the "multiple type" described in the previous section.

The change from the "laminar" to the "turbulent" regime is associated with bubble coalescence. In both regimes the geometry of the vapor phase is more or less spherical.

At higher gas flow rates bubbles interact and form swirling vapor columns which in turn can interact to form large vapor slugs. At these high gas (vapor) flow rates the geometry of the vapor phase in the vicinity of the plate is columnar.

Figs. 5, 6 and 7* show this change of the geometry of the vapor phase with increasing heat flux density (increasing vapor flow rates) in nucleate pool boiling of water at atmospheric pressure.† This change of vapor geometry was first stressed and discussed by Zuber and Tribus [7] and by Zuber [36], it is emphasized here again. The reader should note the presence of single, isolated, bubbles at low heat flux rates and their complete absence at high heat-transfer rates. These pictures clearly indicate that the so called "vapor–liquid exchange mechanism" which is based on the "pumping" action of single bubbles and which was advanced as the mechanism for nucleate boiling is misleading and incorrect at high heat-transfer rates.

2.3. *Formulation of the fluid flow problem*

In two-phase flow systems the flow is a function of the hold-up, i.e. of the fraction of volume occupied by the dispersed phase. An analysis of a batch two-phase process is concerned therefore with determining the relation between the the hold-up and the volumetric flow rate of the vapor phase, Q_v, i.e. the superficial velocity of the vapor, U_{sv}, since

$$\frac{Q_v}{A} = U_{sv}. \tag{11}$$

Such relations have been derived in [59] and [63] for the "laminar" region and in [63] for the turbulent region. For the "laminar" region the superficial vapor velocity U_{sv} is related to the hold-up by

$$U_{sv} = U_t\,\alpha(1-\alpha) \tag{12}$$

whereas in the "turbulent" region it is given by

$$U_{sv} = U_t\,\frac{\alpha}{1-\alpha}. \tag{13}$$

In both expressions U_t is the terminal velocity of a single bubble in an infinite medium and it is given by either (6) or by other expressions given in the literature [46].

The true velocity of rise of the vapor phase, U_v, can be obtained from (12) and (13) by recalling that, by definition, U_v is related to the superficial velocity, U_{sv}, by

$$U_v = \frac{U_{sv}}{\alpha}. \tag{14}$$

Thus, from (12), (13) and (14) the vapor velocity in the "laminar" region is given by

$$U_v = U_t\,(1-\alpha) \tag{15}$$

and in the "turbulent" region it is given by

$$U_v = \frac{U_t}{1-\alpha}. \tag{16}$$

In order to make use of these expressions in the present problem it is necessary to express α in terms of quantities that are measured and

* These photographs were taken by Dr. R. F. Gaertner of the Research Laboratory of the General Electric Company, Schenectady, N.Y. They are part of a study reported in [64] and [65]. The author is indebted to Dr. Gaertner for giving him these photographs as well as for many stimulating discussions.

† Note that the critical heat flux for this condition is approximately equal to 400 000 Btu/h ft^2.

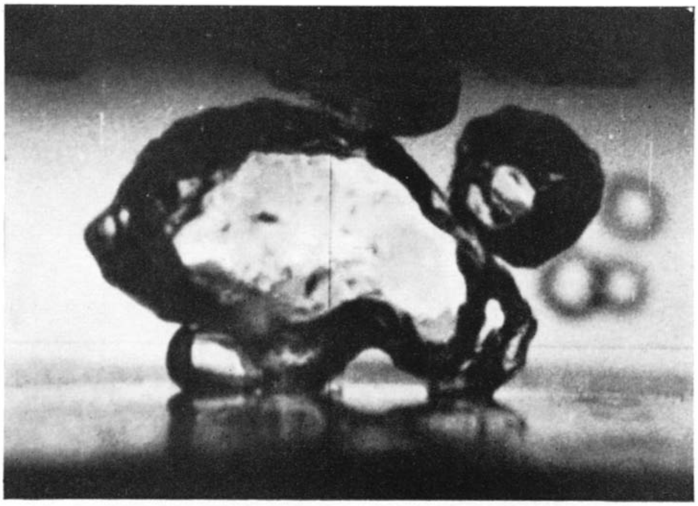


FIG. 4. This photograph was obtained by Mr. R. Semmeria from the Commissariat à l'Énergie Atomique, Grenoble, France. It shows the interaction of two bubbles and the formation of a vapor patch. The reader should note the two stems which attach the patch to the surface.

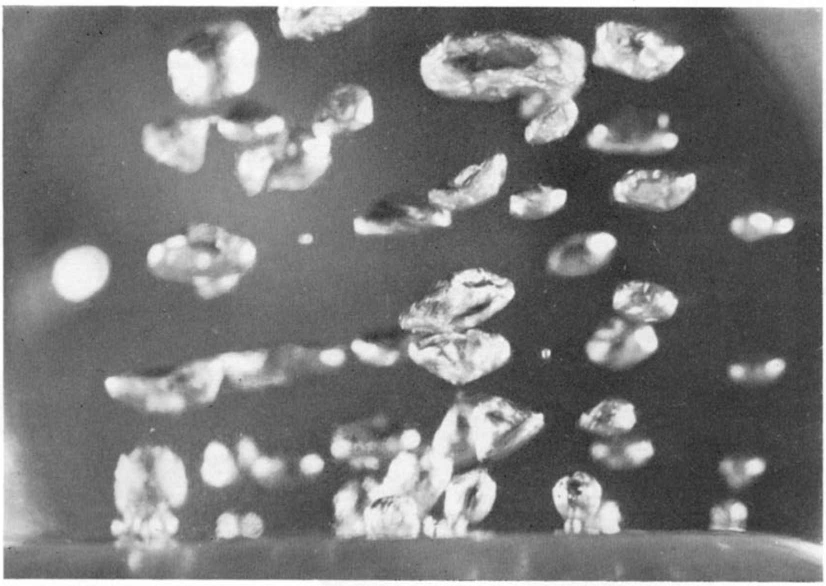


FIG. 5. This photograph was obtained by Dr. R. F. Gaertner from the G.E. Research Laboratory, Schenectady, New York. It shows the region of isolated bubbles. [water: 1 atm, $\dot{Q}/A = 14\,000$ Btu/h ft^2, $T_w - T_s = 18{\cdot}4$°F, Copper surface $d = 2$ in].

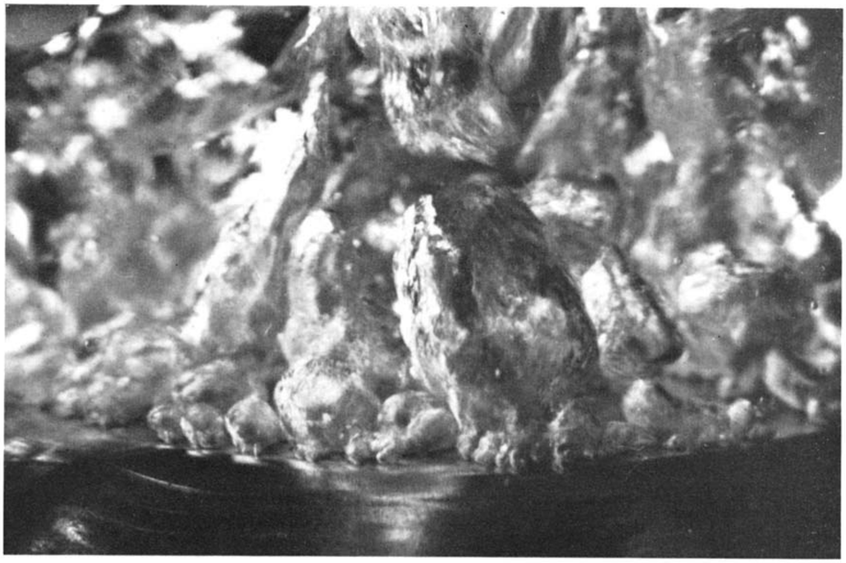


FIG. 6. This photograph was obtained by Dr. R. F. Gaertner from the G.E. Research Laboratory, Schenectady, New York. It shows the region of continuous vapor columns and vapor patches (the region of interference). The reader should note the absence of single bubbles and the numerous stems which attach the columns to the heating surface [water: 1 atm, $\dot{Q}/A = 90\,300$ Btu/h ft^2, $T_w - T_s = 33{\cdot}3\,^\circ$F].

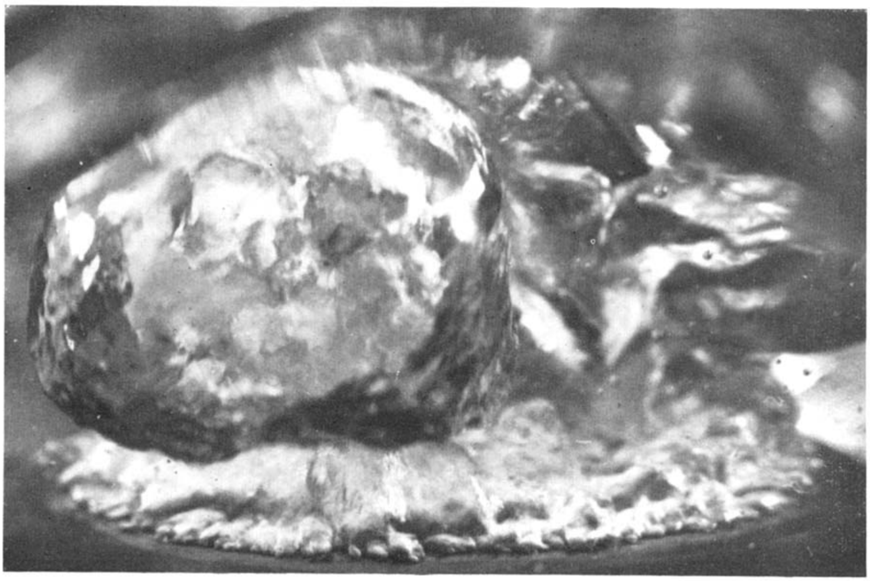


FIG. 7. This photograph was taken by Dr. R. F. Gaertner from the G.E. Research Laboratory, Schenectady, New York. It shows the interaction of vapor columns and the formation of large slugs of vapor [water: 1 atm, $\dot{Q}/A = 204\,700$ Btu/h ft^2, $T_w - T_s = 29{\cdot}8\,^\circ$F].

reported in the literature on nucleate boiling; these quantities are bubble population, bubble diameters and frequencies. To accomplish this we make use of the vapor hold-up expressions derived for analysis of unit operation apparatus [66] and applied to boiling in [11].

For a bubble population density, n/A, a bubble rise velocity, U_v, and a frequency of bubble emission f, the volume of space available to one bubble is AU_v/nf. For bubbles of diameter D_b the vapor hold-up is then by definition

$$\alpha = \frac{\pi}{6} D_b^3 \frac{n}{A} \frac{f}{U_v}. \tag{17}$$

Substituting (15) and (16) in (17) we obtain respectively for the "laminar" regime:

$$\alpha(1-\alpha) = \frac{n}{A} \frac{\pi}{6} \frac{D_b^3 f}{U_t} \tag{18}$$

and for the "turbulent" regime

$$\frac{\alpha}{1-\alpha} = \frac{n}{A} \frac{\pi}{6} \frac{D_b^3 f}{U_t}. \tag{19}$$

Defining the dimensionless group B by:

$$B = \frac{n}{A} \frac{\pi}{6} \frac{D_b^3 f}{U_t} \tag{20}$$

and solving for α we obtain from (18) for the "laminar" regime:

$$\alpha = \tfrac{1}{2} - \sqrt{(\tfrac{1}{4} - B)} \tag{21}$$

where the negative sign was chosen because $\alpha = 0$ when n/A is zero. Similarly solving for α we obtain from (19) for the "turbulent" regime

$$\alpha = \frac{B}{1+B}. \tag{22}$$

In the "laminar" region (21) indicates that the maximum hold-up is

$$\alpha = \tfrac{1}{2} \tag{23}$$

and it occurs when

$$B = \frac{n}{A} \frac{\pi}{6} \frac{D_b^3 f}{U_t} = \tfrac{1}{4}. \tag{24}$$

(24) gives a relation between the bubble population, the bubble diameter at break-off (i.e. at departure), the frequency and the terminal velocity at the maximum hold-up in the "laminar" regime. If $D_b \,.\, f/U_t$ is unity (this corresponds to the maximum possible frequency in the "laminar" regime) then the bubble population which corresponds to the maximum hold-up in the "laminar" bubbling regime is:

$$\frac{n}{A} D_b^2 = \frac{3}{2\pi}. \tag{25}$$

An average bubble spacing was given in [8] by

$$s = \left[\frac{A}{n}\right]^{1/2}. \tag{26}$$

Substituting (25) in (26) we obtain the average bubble spacing at maximum hold-up in the "laminar" bubbling regime, thus:

$$s = \left[\frac{2\pi}{3}\right]^{1/2} D_b = 1{\cdot}44\, D_b. \tag{27}$$

Bubbles will touch each other when

$$s = D_b \tag{28}$$

i.e. when

$$\frac{n}{A} D_b^2 = 1 \tag{29}$$

which is the condition for static interaction.

In the "turbulent" regime (22) indicates that as B increases the hold-up tends to unity. However, if the bubble population increases such that static interaction takes place, i.e. so that (29) is satisfied, and if $D_b \,.\, f/U_t$ is equal to unity then (20) and (22) give for the "turbulent" region

$$\alpha = \frac{\pi}{6+\pi} = 0{\cdot}344. \tag{30}$$

When the bubble population density is small (20), (21) and (22) show that in both the "laminar" and in the turbulent" regime the hold-up can be approximated by

$$\alpha \approx B = \frac{n}{A} \frac{\pi}{6} D_b^2 \frac{D_b \,.\, f}{U_t}. \tag{31}$$

(4), (5), (6) and (7) can be used to express B in terms of the superheat temperature difference, bubble diameter at break-off and terminal velocity, thus:

$$B = \frac{n}{A}\frac{\pi}{6}\frac{\{0{\cdot}021\,\theta\,[\sigma/g(\rho_L - \rho_v)]^{1/2}\}^3}{t_d + t_b} \times \frac{1}{1{\cdot}18\,[\sigma g(\rho_L - \rho_v)/\rho_L{}^2]^{1/4}} \quad (32)$$

or

$$B = \frac{n}{A}\frac{8}{3}\left[\frac{(T_w - T_s)c\rho_L}{\rho_v h_{fg}}\right]^2 \frac{a}{1 + t_d/t_b} \times \frac{0{\cdot}021\,\theta\,[\sigma/g(\rho_L - \rho_v)]^{1/2}}{1{\cdot}18\,[\sigma g(\rho_L - \rho_v)/\rho_L{}^2]^{1/4}}. \quad (33)$$

When the delay time t_d becomes much shorter than the bubble break-off time t_b, then (33) becomes

$$B = \frac{n}{A}\frac{8}{3}\left[\frac{(T_w - T_s)c\rho_L}{\rho_v h_{fg}}\right]^2 \frac{aD_{bF}}{U_t} \quad (34)$$

where D_b and U_t are given by (5) and (6). Similarly if $D_b\,.f/U_t$ then (32) simplifies to

$$B = \frac{n}{A}\frac{\pi}{6}D_{bF}^2. \quad (35)$$

(21), (22), (32) and (33) or their simplified form i.e. (31), (34) and (35) give the hold-up as function of quantities that are measured and reported in the literature; these will be used in the sections that follow to formulate the heat transfer problem.

3 THE HEAT-TRANSFER PROBLEM

During the growth and the departure, a bubble displaces liquid and entrains liquid in its wake. The source flow and the wake flow associated with the growth and departure are shown on Figs. 1a, b. The flow oscillations induce large temperature oscillations in the liquid film adjacent to the heating surface and in the surface itself [5, 11, 67]. The heat-transfer rates in nucleate boiling were therefore attributed, since the systematic investigations of Jakob and co-workers [22, 23, 38, 16, 17], to these bubble induced flows. It is of interest to quote, from [16] and [17] the description of the flow and of the heat-transfer process: "Strong forward and backward flows must occur in the vicinity of a growing and departing bubble; it is even possible that in between neighboring bubble columns downward flowing liquid streams impinge on the surface, whereby, according to Schmidt, Schurig and Sellschop's measurements, the heat transfer becomes extremely high" [17] . . . . "When vapor bubbles rise in large numbers the forced convection induced by the vaporization process becomes important" [17]. "When bubble columns become numerous and evenly distributed over the surface it appears visually that a water circulation is formed whereby water rises together with vapor bubbles, flows downwards at other places and flows essentially horizontally over the heating surface" [16]. "If the total heat transfer is considered as the sum of such local transfers, then the heat transfer coefficient should be independent of the dimensions of the experimental surface" [17].

By comparing Jakob's description of nucleate pool boiling from a horizontal surface and Townsend's description of the flow regime in turbulent natural convection (given in Appendix C), it appears that the flow regimes in these two heat-transfer processes are rather similar. The flow through the conduction layer, the flow through the "up-draught", the localized nature and the maintenance of the "up-draught sites" described by Townsend, are similar to the flow parallel to the surface and to the flow associated with a bubble column rising above a nucleating site as described by Jakob. Indeed the flow depicted on Fig. 1 can be looked upon as an "up-draught" induced by the bubble motion. The temperature fluctuations in the regions of "activity" described by Townsend, and which are characteristic of the convective processes arising near the rigid boundary, are not dissimilar with the temperature fluctuations observed in nucleate boiling in the vicinity of the heating surface.

In view of the foregoing it appears desirable to formulate the heat-transfer problem by considering turbulent natural convection.

3.1. *Formulation of the heat-transfer problem*

The problem of natural convection from horizontal surfaces was recently investigated experimentally and analytically by Malkus [18, 19] and by Thomas and Townsend [20, 21]. The experimental results confirmed that at high

Rayleigh numbers the Nusselt modulus was proportional to the cube root of the Rayleigh number, thus:

$$\frac{hd}{k} = \text{const.}\left[\frac{g}{\nu a}\beta \Delta T d^3\right]^{1/3} \qquad (36)$$

where

$$\beta = \frac{\rho_\infty - \rho_w}{\rho_\infty}.$$

In order to predict the heat transport and turbulent spectrum from a theoretical model of turbulent phenomena Malkus advanced the proposition that the flow adjusts itself in such a way as to transfer the maximum amount of heat compatible with the boundary conditions. He related the transport properties of the fully turbulent flow to the neutrally stable disturbances of the corresponding laminar flow. Without introducing experimental constants, Malkus succeeded in predicting the mean velocity distribution for the turbulent flow in a two-dimensional channel with reasonable accuracy. For the average-square-velocity fluctuation he found the following expression:

$$\bar{\bar{v}}_L^2 = \frac{1}{9}\frac{g\beta\Delta T da}{\nu}. \qquad (37)$$

In natural convection, as the temperature of the surface increases, the density of the liquid adjacent to it decreases, whereby buoyant forces induce a flow. This effect is reflected in the density difference term in (36). In nucleate boiling the density of the mixture decreases even further because of the vapor formation at the surface. Denoting the vapor hold-up at the heating surface by: a_w the density of the two-phase mixture adjacent to the heating surface can be expressed as

$$\rho_{mw} = (1 - a_w)\rho_{Lw} + a_w\rho_v. \qquad (38)$$

The difference in density becomes then

$$\rho_{L\infty} - \rho_{mw} = \rho_{L\infty} - \rho_{Lw} + a_w(\rho_{Lw} - \rho_v) \qquad (39)$$

and introducing the coefficient of thermal expansion, it follows that

$$\frac{\rho_{L\infty} - \rho_{mw}}{\rho_{L\infty}} = \beta\Delta T + a_w\frac{\Delta\rho}{\rho_{L\infty}} \qquad (40)$$

where

$$\Delta\rho = \rho_{Lw} - \rho_v.$$

The heat-transfer coefficient and the turbulent velocity fluctuation can be expressed in terms of the vapor fraction by introducing (40) in (36) and (37) respectively, thus

$$\frac{hd}{k} = \text{const.}\left[\frac{gd^3}{\nu a}\left(\beta\Delta T + a\frac{\Delta\rho}{\rho_{L\infty}}\right)\right]^{1/3} \qquad (41)$$

and

$$\bar{\bar{v}}_L^2 = \tfrac{1}{9}\frac{gad}{\nu}\left(\beta\Delta T + a_w\frac{\Delta\rho}{\rho_{L\infty}}\right). \qquad (42)$$

The values of the hold-up a_w to be inserted in (41) and (42) are given by (21) or (22) and (32) or (33). However, the agreement with experimental data (shown in the sections that follow) seem to justify the use of the simplified forms, i.e. of (31) and (34) or (35). Substituting (31) and (34) in (41) and (42) results in the approximations

$$\frac{hd}{k} = \text{const.}\left\{\frac{gd^3}{\nu a}\left[\beta\Delta T + \frac{n}{A}\frac{8}{3}\left(\frac{\Delta T c\rho_L}{\rho_v h_{fg}}\right)^2\frac{aD_{bF}}{U_t}\frac{\Delta\rho}{\rho_{L\infty}}\right]\right\}^{1/3} \qquad (43)$$

$$\bar{\bar{v}}_L^2 = \tfrac{1}{9}\frac{gad}{\nu} \times \left[\beta\Delta T + \frac{n}{A}\frac{8}{3}\left(\frac{\Delta T c\rho_L}{\rho_v h_{fg}}\right)^2\frac{aD_{bF}}{U_t}\frac{\Delta\rho}{\rho_{L\infty}}\right]. \qquad (44)$$

While inserting (31) and (35) in (41) and (42) results in the approximations

$$\frac{hd}{k} = \text{const.}\left[\frac{gd^3}{\nu a}\left(\beta\Delta T + \frac{n}{A}\frac{\pi}{6}D_{bF}^2\frac{\Delta\rho}{\rho_{L\infty}}\right)^{1/3} \qquad (45)$$

and

$$\bar{\bar{v}}_L^2 = \tfrac{1}{9}\frac{gad}{\nu}\left(\beta\Delta T + \frac{n}{A}\frac{\pi}{6}D_{bF}^2\frac{\Delta\rho}{\rho_{L\infty}}\right) \qquad (46)$$

where D_{bF} and U_t are given by (5) and (6).

The preceding results are compared with experimental data in the section that follows. It is important to remark here that the volumetric vapor coefficient a_w which appears in the preceding equations was evaluated for conditions existing close to the surface and it is not identical with the average void coefficient for the entire boiling mixture. At low heat flux (to be discussed later) the vaporization process is not completed at the surface but proceeds with bubbles still growing while rising [23]. Consequently α_w is smaller than the average void coefficient for the entire mixture.

3.2. *Comparison with experimental data*

Before comparing the results derived in Section 3.1 with experimental data, several preliminary observations can be made.

First we note that when $D_b^3 . f$ remains constant as reported for the experiments of [11] and [12] then (30) indicates that the heat-transfer coefficient is proportional to

$$h \sim n^{1/3}$$

which is in agreement with the results of [11, 12] and [13].

When the gravitational field is decreased, (43), (5) and (6) indicate a decrease of the heat-transfer coefficient, a prediction which is in agreement with the experimental results of Siegel and Usiskin [68]. It is interesting to note also that when the value of g is increased the same equations indicate that (for the same bubble population) the effect of vaporization relative to the effect of natural convection ($\beta \Delta T$) decreases. This result is in qualitative agreement with the observations of Merte and Clark [69].

When the effect of the natural convection is neglected compared to the effect of vaporization, (43) indicates that the heat flux density is proportional to

$$\frac{\dot{Q}}{A} \sim (T_w - T_s)^{5/3} \left(\frac{n}{A}\right)^{1/3} \tag{47}$$

whence a two-parameter expression of the form (3) results. It was noted in the introduction that Yamagata and Nishikawa [11] found, from experiments performed by varying the surface tension, that the exponents in (3) were given by $b = 3/2$ and $c = 1/4$. Equation (43) i.e. (47) predicts $b = 5/3$ and $c = 1/3$ for these exponents, these values differ also from the values reported in [8] and [15]. The comparison of the results predicted by the present analysis with the available data (to be shown in the following) indicates that the exponents predicted by (47) are probably the more correct ones.

Equation (47) indicates also that the effect of increasing the bubble population while maintaining the same heat flux density results in a decrease of the superheat temperature difference according to

$$\left.\frac{d[\ln (T_w - T_s)]}{d[\ln (n/A)]}\right|_{\dot{Q}/A = \text{const.}} = -\tfrac{1}{5}. \tag{48}$$

A value for this expression obtained from experiments and equal to $-1/6$ was reported in [11, 12]. It should be noted that the scatter of the data is such that the results can be approximated also by an exponent equal to $-1/8$ as shown in [8].

It can be seen from (41) that the effect of vaporization on the heat-transfer coefficient is reflected in terms of one parameter only, i.e. in terms of the vapor void coefficient α_w. If α_w is not measured, this effect can be expressed in terms of two parameters: the bubble population density, and the liquid superheat temperature difference or the bubble diameter at departure [cf. (43) and (45)]. References [5, 11, 12, 13, 14] report simultaneously recorded data on the bubble population density n/A, liquid superheat $(T_w - T_s)$ and the heat transfer coefficient h. These experiments were performed with a variety of liquids and for various surface conditions and permit therefore a quantitative evaluation of the analysis.

Figs. 8a, b are reproduced from [5] and show the experimental data of Courty and Foust for n-pentane in pool boiling from a horizontal nickel surface for two surface polishes. Fig. 8a indicates that a surface with larger nucleating cavities (a rougher surface) requires lower values of $T_w - T_s$ than a smooth surface. Fig. 8b indicates clearly that in nucleate boiling h is not a single valued function of the temperature but depends upon both the surface nucleating characteristics (surface conditions) and the superheat temperature difference. Inserting the values of n/A taken from Fig. 8a into (45) permits an estimate of the heat-transfer coefficient; the predicted values of h thus computed are shown on Fig. 8b as heavy lines. The predicted values shown on Fig. 8b have been computed only in the range of the experimental data shown on

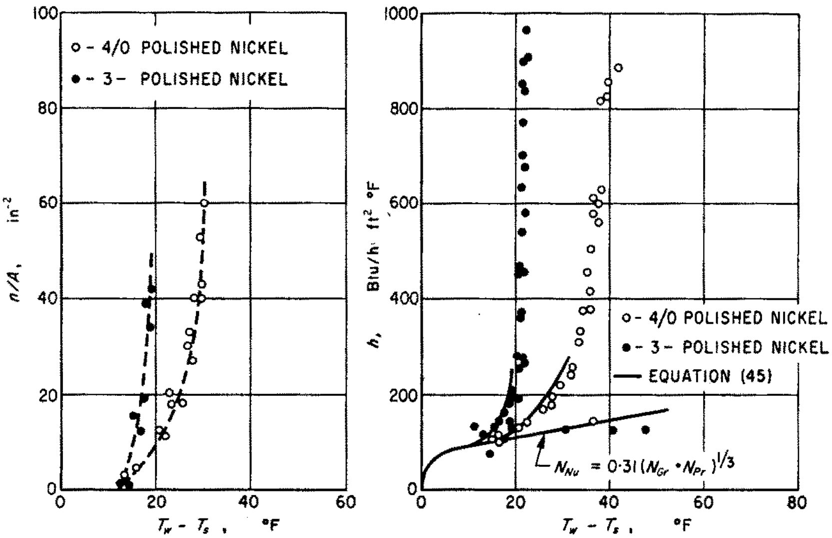


FIG. 8. The effect of a different amount of roughness on the bubble population density, heat-transfer coefficient, and liquid superheat temperature difference for n-pentane in nucleate pool boiling. The experimental data are taken from reference [5]. The values predicted by equation (45) are plotted also.

Fig. 8a. It is important to note here that in the natural convection regime (with no boiling taking place, $\alpha = 0$), in order to obtain an agreement with the experimental data (examined in this paper), it was necessary to take the value of 0·31 for the constant in (36) instead of 0·16 as reported by Jakob. It came as a surprise to the writer that the effect of natural convection was never evaluated in papers reporting experimental results. The reason for the values of 0·31 instead of 0·16 is not entirely clear; it remains a task for future experimental investigations to clarify this point. In this paper the value of 0·31 was taken for all computations and comparisons. Fig. 8b indicates that the agreement of values predicted by (45) with the experimental data is satisfactory.

In Table 1 the heat-transfer coefficient predicted by (43) or (45) are compared with the experimental data of Kurihara and Myers [13] for acetone, CCl_4 and CS_2 in pool boiling at atmospheric pressure from a horizontal surface.

On Fig. 9 the heat-transfer coefficient predicted by (43) or (45) is compared with experimental data reported in [11, 13] and [14] for water in nucleate pool boiling at atmospheric pressure from a horizontal surface. The agreement appears as satisfactory as the scatter of the experimental data permits.

Table 1

Liquid	$T_w - T_s$ (degF)	n/A (ft^{-2})	h (Btu/h ft² degF)	
	By tests	By tests	By tests	By equations (43) and (45)
Acetone	15·1	163	105	160
	21·4	550	150	194
	21·8	715	155	200
	25·9	1430	241	232
	26·5	1610	229	248
CS_2	16·2	82	134	144
	22·5	600	169	170
	25·7	1510	257	210
CCl_4	13·9	408	107	131
	20·4	1020	127	162
	22·9	2450	178	184

Fig. 9 shows that for the conditions of Gaertner and Westwater [14] there occurs at approximately $h = 1400$ Btu/h ft² degF, a rather sharp change in the slope of the curve of the predicted heat-transfer coefficient, i.e. for superheat temperatures $T_W - T_S$ exceeding 41°F the predicted value of h decreases with increasing $(T_W - T_S)$ (dashed line). Furthermore, a sharp

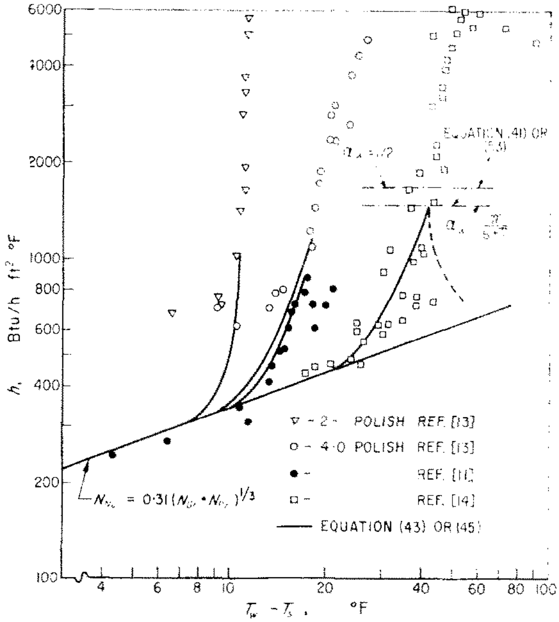


FIG. 9. The comparison of predicted heat-transfer coefficient in the regime of isolated bubbles with experimental data for water at 1 atm and various surface conditions.

change of the trend of heat-transfer coefficient is exhibited also by the experimental data plotted on Fig. 10; at approximately $h = 1450$ Btu/h ft² degF, the value of D_b starts decreasing with increasing h. It is this decrease of D_b, which causes a decrease of the predicted value of the heat-transfer coefficient. The abrupt changes of D_b and of h are due to changes in flow regimes and will be taken up in the sections that follow.

In their fundamental experimental investigation of boiling, Yamagata and Nishikawa [11], in addition to simultaneously measuring the values of n/A, $(T_w - T_s)$, and of h, determined also the average liquid velocity in the thermal boundary layer adjacent to the heating surface. This velocity was determined by photographing the image of refracted light rays passing through the boiling liquid. Fig. 11 which is reproduced from [11] shows the statistical mean value of this liquid velocity as function of the number of bubble columns (active nucleating sites on a surface area of 78·6 cm²). The average turbulent velocity fluctuations predicted by (46) is also plotted on Fig. 11. The diameter of the surface ($d = 10$ cm) was taken for the characteristic length appearing in (46). The agreement is significant, especially in view of the fact that no experimental constants appear in (46).

It appears from the results and comparisons presented in this section that the values predicted from the analysis are in qualitative and in quantitative agreement with the experimental data presently available.

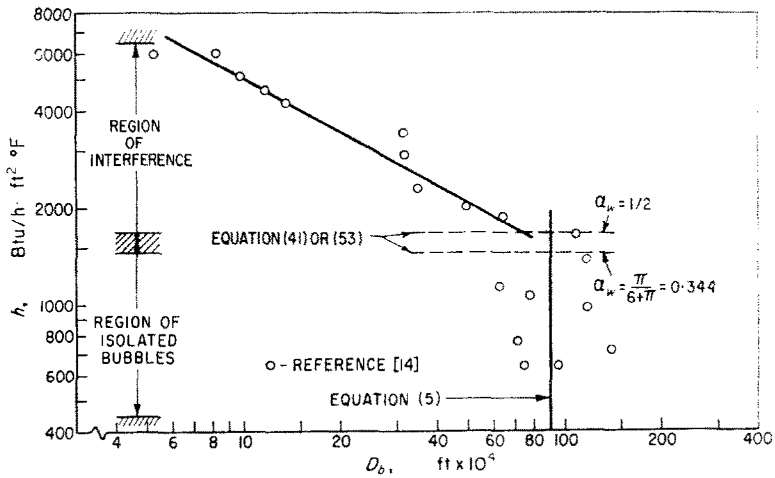


FIG. 10. The experimentally determined variation of bubble diameter at departure with heat-transfer coefficient reported by Gaertner and Westwater [14] for water at 1 atm. This figure shows also the domains for the two boiling regimes and the upper limit for the heat-transfer coefficient in the regime of isolated bubbles predicted by the present analysis.

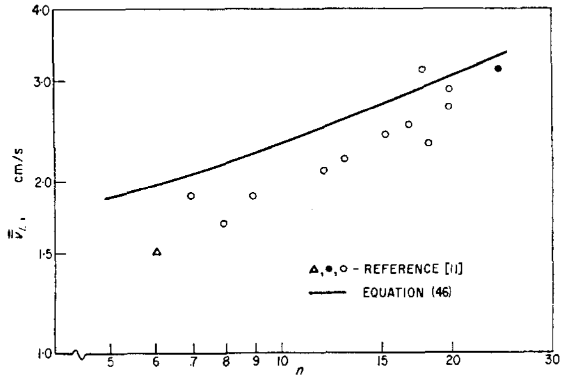


FIG. 11. Comparison of predicted values of the average turbulent velocity fluctuation with experimental data of Yamagata and Nishikawa [11].

4. DISCUSSION OF RESULTS

4.1. *Hydrodynamic similarity and the vapor void fraction*

Figs. 8b and 9 clearly indicate that an expression of the form (1) cannot correlate the data with only one value of the constant and of the exponent. This fact implies that hydrodynamic similarity is not preserved when an equation of the form (1) is used. A similar statement applies also to (2) because Figs. 8a, b indicate that instead of plotting h against $(T_w - T_s)$, a plot of h against n/A would have resulted also in two distinct curves.

It appears therefore, that neither $(T_w - T_s)$ or n/A taken alone are sufficient to ensure hydrodynamic similarity. The agreement of predicted values with experimental data as shown on Figs. 8b and 9 indicates, however, that the hydrodynamic similarity will be preserved when the volumetric vapor coefficient a_w is taken into account.

The agreement of predicted values with experimental data indicates also that, in the range where the present analysis can be applied (discussed in the next section) the volumetric vapor fraction a_w (evaluated at the heating surface) in nucleate pool boiling of liquids at saturation temperature can be expressed as a function of the heat-transfer coefficient, thus:

$$a_w = \frac{va}{g}\left(\frac{h}{0{\cdot}31\,k}\right)^3 - \beta \Delta T. \qquad (49)$$

4.2. *The region of isolated bubbles and the interference of bubbles*

It was observed in [8] that for values of h up to approximately 1450 Btu/h ft² degF the mean value of D_b can be approximated by the equation of Fritz, i.e. by (5) with $\theta = 50°$ (Fig. 10). It was also noted that the maximum bubble population density corresponds to the condition when bubbles touch each other and it is given by (29). For water at atmospheric pressure taking $D_{bF} = 2{\cdot}8 \times 10^{-1}$ cm (i.e. $D_b = 92 \times 10^{-4}$ ft), this population density corresponds approximately to 13 bubbles/cm², i.e. to 11 800 bubbles/ft². Fig. 12 shows a plot of D_b against n/A for the experimental data of Gaertner and Westwater [14]. It can be seen that the bubble population exceeds, by far, the value of 11 800 bubbles/ft². In this case also it can be seen that before this maximum value is reached there occurs at approximately $n/A = 3000$ bubbles/ft², a change in the D_b against n/A relation. For population densities smaller than 3000 bubbles/ft² the bubble diameter is independent of the bubble population density and can be predicted by the Fritz equation. It is for this reason that the region for which $D_{bF} \neq f(n/A)$ was referred to as "the region of isolated bubbles". It can be also seen from Fig. 12 that for population densities larger than 6000 bubbles/ft² the bubble diameter depends upon the population (D_b decreasing with increasing n/A). This region is referred here as the "region of interference" or the "region of continuous vapor columns and patches". In this region the diameter corresponds to the diameter of the vapor stems which keep the columns and patches attached to the surface (Figs. 4 and 6). (This aspect of the problem is analysed in [65]).

An average bubble spacing was defined in [8] and it is given by (26). The average spacing corresponding to the population density of 3000 bubbles/ft² is $s = 181 \times 40^{-4}$ ft. It can be seen from Fig. 12 that this value corresponds to a spacing equal to

$$s_c = 2D_{bF} = 0{\cdot}042\,\theta\left[\frac{\sigma}{g(\rho_L - \rho_v)}\right]^{1/2} \qquad (50)$$

with $\theta = 50°$ approximately. Consequently the region of isolated bubbles was defined in [8] as the region for which $s > 2D_{bF}$. More recently Hsu and Graham [33] proposed a criterion

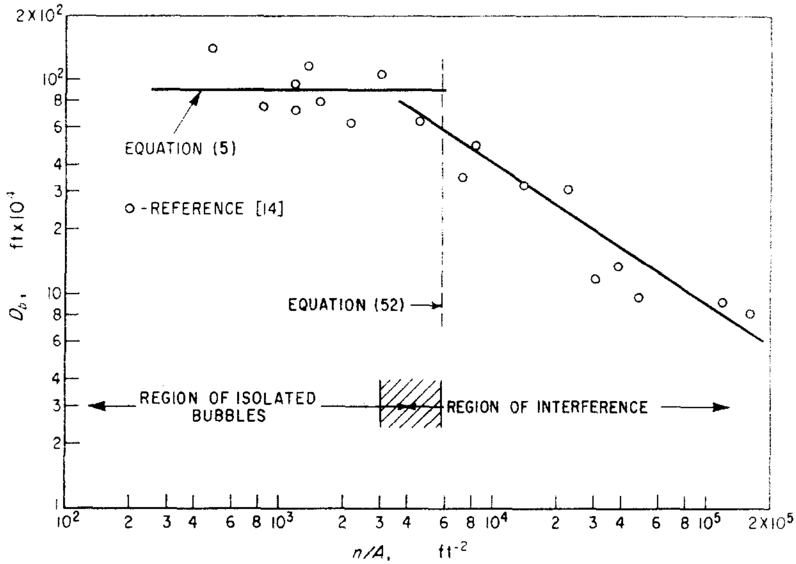


FIG. 12. The experimentally determined variation of bubble diameter at departure with the bubble population density reported by Gaertner and Westwater for water at 1 atm. This figure shows also the two regions in nucleate boiling and the predicted bubble population density at the transition between the two regions.

$s_c = 1{\cdot}85 \times D_{bF}$ for this domain, whereas Gaertner [58] derived $s_c = 1{\cdot}5\ D_{bF}$ by considering a Poisson distribution.

Equation (27) gives the spacing that corresponds to the maximum hold-up, i.e. vapor volumetric fraction in the "laminar" regime, i.e.

$$s_c = \left(\frac{2\pi}{3}\right)^{1/2} D_{bF} = 1{\cdot}44\ D_{bF}. \qquad (51)$$

This value compares favorably with the results of Gaertner [58].

The bubble population density which corresponds to this spacing is given by (25). Inserting the value of $D_{bF} = 90 \times 10^{-4}$ ft into (25) the predicted bubble density at the upper limit of the region of isolated bubble (if this limit corresponds to the limit of the "laminar" bubbling regime) is

$$\frac{n}{A} = \frac{3}{2\pi}\frac{1}{D_{bF}^2} = 5900\ \text{ft}^{-2} \qquad (52)$$

which is in satisfactory agreement with the data shown on Fig. (12).

4.3. *The limiting value of the heat-transfer coefficient in the region of isolated bubbles*

Fig. 10 shows a change in the h against D_b relation which takes place at approximately $h = 1450$ Btu/h ft² degF. It was observed in [8] that this change probably corresponds to the inflexion point of the h against $(T_w - T_s)$ curve (Fig. 9). For values of h smaller than 1450 Btu/h ft² degF, D_b is constant and it is independent of h, this region corresponds to the region of isolated bubbles. For values of h larger than 1450 Btu/h ft² degF, D_b is a function of h; this region corresponds to the region in which D_b is a function of the bubble population (Fig. 12).

Fig. 9 indicates that in the region of isolated bubbles the heat-transfer coefficient predicted by (41) is in satisfactory agreement with experimental data. It can be seen from (41) that, for the assumed heat-transfer mechanism, the maximum value of h will correspond to the maximum value of the vapor volumetric fraction α_w. Equation (23) gives the maximum value of α_w in the "laminar" bubbling regime, whereas (30) is a reasonably limiting value (limited by the static bubble interaction) of α_w in the "turbulent" bubbling regime. For water at atmospheric pressure neglecting β (β is of the order of $7{\cdot}5 \times 10^{-4}$ 1/°C) and taking the value of the constant equal to 0·31, the approximate form of (41) then becomes

$$h = 0{\cdot}31\, k \left(\frac{g}{\nu a}\, a_w\, \frac{\Delta\rho}{\rho_{L\infty}}\right)^{1/3} \tag{53}$$

which predicts for the "laminar" bubbling regime (with $a_w = 0{\cdot}5$):

$$h = 1670 \text{ Btu/h ft}^2 \text{ degF}$$

and for the "turbulent" bubbling regime (with $a_w = \pi/6 + \pi = 0{\cdot}344$)

$$h = 1470 \text{ Btu/h ft}^2 \text{ degF}.$$

These predicted limiting values for the heat-transfer coefficient in the region of isolated bubbles are shown in Figs. 9 and 10. The agreement with experimental data is satisfactory.

4.4. *The limiting value of the heat flux density in the region of isolated bubbles*

By making use of the results given in Section 2.3 it is possible to predict also the maximum value of the heat flux density in the region of isolated bubbles.

In the "laminar" bubbling regime the maximum hold-up is given by (23); inserting this value in (12) the maximum value of the superficial vapor velocity in the "laminar" bubbling regime becomes

$$U_{sv} = \tfrac{1}{4} U_t \tag{54}$$

where U_t is given by (6). For liquids at saturation temperature the energy transferred from the surface is used for vaporization. It follows then from an energy balance that

$$\frac{\dot{Q}}{A} = \rho_v h_{fg} U_{sv}. \tag{55}$$

Substituting (54) and (6) in (55) we obtain the heat flux density which corresponds to the maximum hold up in the "laminar" bubbling regime, thus:

$$\frac{\dot{Q}}{A} = \rho_v h_{fg}\, \frac{1{\cdot}53}{4} \left[\frac{\sigma g(\rho_L - \rho_v)}{\rho_L^2}\right]^{1/4}. \tag{56}$$

(We have used the value of 1·53 for the constant in (6) as recommended in [45].)

For the "turbulent" bubbling regime using the same procedure we obtain from (30) and (13)

$$U_{sv} = \frac{\pi}{6}\, U_t. \tag{57}$$

Substituting (57) and (6) in (55) we obtain the heat flux density which corresponds to the hold-up value of $a = \pi/(6 + \pi)$ in the "turbulent" bubbling regime, thus

$$\frac{\dot{Q}}{A} = \rho_v h_{fg}\, \frac{\pi}{6}\, 1{\cdot}53 \left[\frac{\sigma g(\rho_L - \rho_v)}{\rho_L^2}\right]^{1/4}. \tag{58}$$

Equations of a similar form but with different values for the numerical constant and based on different models and arguments were given in [65] and [70].

In Fig. (13) the heat flux density at the upper limit of the region of isolated bubbles predicted by (56) and (58) is compared with experimental data. The agreement is satisfactory here again.

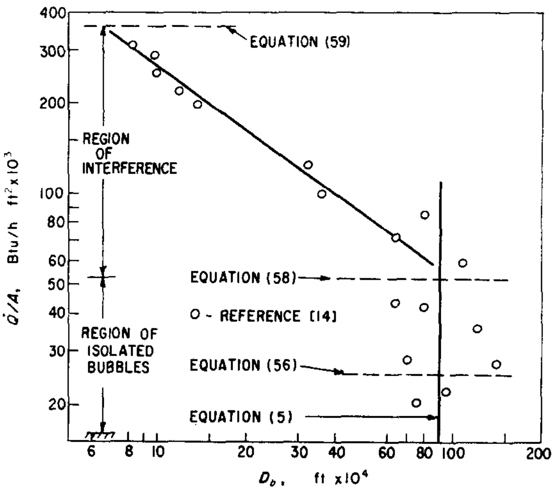


FIG. 13. The experimentally determined variation of bubble diameter at departure with heat flux density reported by Gaertner and Westwater [14] for water at 1 atm. This figure shows also the limits of the two regions predicted by the analysis of this paper.

4.5. *The change in the two-phase flow regimes and in the heat-transfer mechanism*

The results presented in the preceding section show that in the region of isolated bubbles the values predicted by the analysis (i.e. the heat-transfer coefficient, the vapor hold-up, the average turbulent velocity fluctuation, as well as the limiting values of the bubble population, of the bubble spacing, of the heat-transfer coefficient and of the heat flux density) are in quantitative agreement with experimental data. The results show also that in the region in which bubbles interfere with each other such an agreement is not obtained; this is attributed to a change of the heat-transfer mechanism and of the flow regimes caused by the interaction of bubbles. This statement will be elaborated upon in what follows.

It was noted in Section 3 that the flow regime in nucleate pool boiling described by Jakob was in qualitative agreement with Townsend's description of the flow regime in turbulent natural convection from a horizontal surface. The results presented in the preceding sections indicate that a quantitative agreement exists as well. It is important to note that in both cases the heat-transfer process is a consequence of the circulation induced by the "up-draughts". In nucleate pool boiling the flow is upward with a rising bubble column, downward in between bubble columns and more or less horizontal along the surface. If we neglect the effect of latent heat transport it is immaterial, according to this heat-transfer mechanism, whether this circulation is induced by vapor bubbles or by some other means, for example by gas bubbles. Indeed, the experiments of Beatty and coworkers [71] have shown that an electrolytic generation of gas at the heating surface has the same effect on the heat-transfer coefficient as the generation of vapor bubbles. However, their experiments indicated also that there exists an upper limit for the gas-bubble flow agitation effect. It was shown in the preceding sections that, in the region of isolated bubbles, the heat-transfer coefficient has an upper limit.

For the heat-transfer mechanism caused by the "up-draughts" (neglecting the latent heat transport), it is immaterial whether the circulation is induced by vapor bubbles or by some other means. However, it is essential that the circulation be maintained in the vicinity of the heating surface. In nucleate pool boiling (and in gas generation from the surface) such a circulation is maintained by the constant displacement and entrainment of liquid by rising bubbles (Figs. 1b, and 5). It is apparent also that such a circulation can be maintained only if the generation of vapor is intermittent, i.e. in terms of single, distinguishable bubbles. If a rising bubble interferes with the preceding one, coalescing and forming a continuous vapor column (two or more bubble diameters in length), (Figs. 3c, 6 and 7), the intermittent generation of vapor ceases and a more continuous generation starts. The formation of continuous vapor columns will drastically change the displacement and entrainment of the liquid and the liquid circulation in the vicinity of the heating surface. It appears therefore that a change of the circulation and, therefore, of the heat-transfer mechanism should be associated with a change of the mechanism of vapor removal caused by bubble interaction, i.e. due to a change in the two-phase flow pattern in the vicinity of the heating surface.

The distinct difference in the flow pattern for the vapor removal: single bubbles at low heat flux densities and continuous vapor columns and vapor patches at higher flux densities was discussed in some detail in [34, 7, 36] and in Section 2.2 of this paper. Figs. 5, 6 and 7 show this change. The continuous vapor columns and the latent heat transport were indeed used in formulating a conceptual model and deriving an expression for the critical heat flux in nucleate pool boiling [7, 35, 36, 37]. Recent investigations of Stock [10], of Wallis [59] and of Gaertner [64, 65], provide further experimental evidence confirming both the basic difference between the two vapor removal mechanisms and the hydrodynamic character of the critical heat flux.

The results of this paper indicate that in the regime of isolated bubbles the heat transfer is caused by the "up-draughts" and the liquid circulation, whereas the results of [7, 34, 35, 36, 37] indicate that the critical heat flux can be evaluated by considering only the transport of latent energy of vaporization. It is of interest to inquire whether or not the latent heat transport is the dominant mechanism of heat transfer in the entire region of a continuous vapor column and vapor patches, i.e. in the region where bubbles interfere with each other.* The answer is

* This region was called by Zuber [36] "patches of transitional boiling" and by Wallis [59] "patchy boiling".

affirmative although with some qualifications because of insufficient experimental data.

Experimental evidence pertaining to the latent heat transport was recently reported by Moore and Mesler [31] for liquids at saturation temperature and by Bankoff and Mason [32] for sub-cooled liquids. From a detailed study of temperature fluctuations in nucleate boiling Moore and Mesler [31] concluded that, at high heat flux densities, the only mechanism which was consistent with their observation was a removal of heat by rapid vaporization of a micro-layer at the base of the bubble. The usually accepted mechanism of "bubble agitation" could not account for their experimental results. Moore and Mesler noted that "different factors are probably dominant at high heat fluxes than at low" also that the micro-layer vaporization mechanism, i.e. the latent heat transport appears to be the dominant factor at high flux densities.

The dominant pattern of vapor removal at high heat flux densities are continuous columns and vapor patches. Figs. 4 and 6 show that these columns and patches are attached to the surface by many stems. The evaporation (as noted in connection with Fig. 4) takes place at these stems; while the vapor is transmitted through the columns to form large vapor slugs (Figs. 6 and 7). It appears therefore that in the region where bubbles interfere with each other, the dominant mechanism of heat transfer is by latent heat transport caused by rapid vaporization of a pulsating micro-layer at the base of continuous vapor columns or of vapor patches. An analysis based on this model was recently reported by Gaertner [65].

It is of interest to observe, in closing, an additional similarity between turbulent natural convection from a horizontal surface and nucleate pool boiling.

In analysing his data on turbulent natural convection from a horizontal surface Malkus [18, 19] remarked that the most suggestive observation from his experiments was "the apparent lack of dependence of the heat transport on the character of the horizontal motion; almost as though the heat transport was separately determined while the fluid motion adjusted itself to fit the new boundary condition".

A similar observation can be made in connection with nucleate pool boiling from a horizontal surface. In the regime of isolated bubbles as the heat flux is increased (a change in the boundary conditions) the vapor hold-up [i.e. the liquid superheat temperature difference $(T_w - T_s)$, the bubble population n/A, the bubble diameter D_b, the frequency f] and, consequently, the flow field will change in such a way as to accommodate the new boundary conditions. Further increase of the heat flux density brings about a change of the flow regimes from isolated bubbles to continuous vapor columns and patches. Here again the two-phase flow pattern (i.e. the characteristics of the flow field) has changed in order to fit the new boundary conditions and to accommodate the increased vapor generation and transport. In contrast to natural convection, however, in nucleate pool boiling there exists a limit beyond which the flow field cannot adjust itself to fit a change in the boundary conditions. When this limit is reached both the flow field and the boundary condition must change. This limit is imposed on the system by the Helmholtz–Taylor two-phase flow instability (the liquid streams toward the surface and of the vapor streams away from it). It is this two-phase flow instability [34, 7, 35, 36, 37] which brings about the critical heat flux phenomenon in nucleate pool boiling. The upper limit for the heat flux in the regime of continuous vapor columns and patches can be predicted from an equation of the form [34, 7, 35, 36, 37]

$$\frac{\dot{Q}}{A} = \rho_v h_{fg} \frac{\pi}{24} \left[\frac{\sigma g(\rho_L - \rho_v)}{\rho_v^2}\right]^{1/4} \tag{59}$$

This predicted upper limit for the region of interference is shown in Fig. 13. (The thermodynamic properties were evaluated at the saturation temperature.) Thus, the limits of both the region of isolated bubbles and of the region of interference (the domain of continuous vapor columns and patches) can be evaluated.

5. SUMMARY AND CONCLUSIONS

(1) Experimental evidence indicates that nucleate pool boiling from a horizontal surface of liquids at saturation temperature consists of two regions (a) the region of isolated bubbles

and (b) the region of interference. In this paper the vapor removal pattern, the two-phase flow characteristics and the mechanism of heat transfer in the two regions were discussed and analysed.

(a) *The regime of isolated bubbles*

(2) This regime is characterized by the intermittent generation of single distinguishable bubbles (cf. Fig. 5). Bubbles do not interfere with each other. The diameter of a bubble departing from the heating surface is independent of the bubble population density and can be predicted by the equation of Fritz, (cf. Fig. 12).

(3) In this regime Jakob's description of the liquid flow in nucleate pool boiling: upward with rising bubble columns, downward in between bubble columns and approximately horizontally along the surface is similar to Malkus' and Townsend's description of the flow regime (caused by the "up-draught) in turbulent natural convection from a horizontal surface. In both cases the heat transfer is effected by the "up-draught" induced circulations.

(4) It is shown that the same equation which predicts the heat-transfer coefficient in turbulent natural convection can be used in the regime of isolated bubbles if the volumetric vapor fraction (vapor void coefficient) is taken into account in evaluating the mean density of the fluid [cf. equation (41)]. The predicted values are in qualitative and quantitative agreement with available experimental data (cf. Figs. 8, 9, 10). When the effect of thermal expansion of the liquid is neglected a two parameter expression (in terms of the temperature difference $(T_w - T_s)$ and of the bubble population) is obtained [cf. equation (47)].

(5) It is also shown that the same equation, originally derived by Malkus, for predicting the average turbulent velocity fluctuation in natural convection from a horizontal surface can be used in the regime of isolated bubbles if the volumetric vapor fraction is taken into account in evaluating the mean density of the fluid [cf. equation (42)]. Satisfactory agreement with available experimental data is shown (cf. Fig. 11) [note that no empirical constants appear in (46) i.e. in equatiion (42)].

(6) The results indicate that a particular regime of nucleate pool boiling (a two-phase flow problem) can be analysed as a turbulent natural convection problem (a single-phase flow problem) if the vapor volumetric fraction (vapor hold-up) is taken into account in evaluating the density of the medium.

(7) In this regime the vapor void coefficient, i.e. the vapor hold-up can be expressed as a function of the heat-transfer coefficient [cf. equation (49)]. Equations relating the vapor void coefficient (vapor hold-up) to the bubble population density and the liquid superheat temperature are presented also [cf. equations (20), (21), (22), (32), (33)].

(8) It is shown that in the regime of isolated bubbles an upper limit exists for both the value of the vapor hold-up (vapor volumetric fraction) and for the value of the heat-transfer coefficient. Consequently an upper limit exists for the heat-transfer mechanism effected by the "up-draught" circulations.

(9) Equations which predict: (i) the bubble spacing, (ii) the bubble population, (iii) the vapor hold-up, (iv) the heat-transfer coefficient and (v) the heat flux density at the upper limit of the region of isolated bubbles have been presented. The predicted values are in agreement with available experimental data (cf. Figs. 10, 11, and 13). Thus, the conditions leading to the change from the region of isolated bubbles to the region of interference can be evaluated.

(b) *The region of interference*

(10) In the region of interference bubbles interfere with each other and form continuous vapor columns and patches (cf. Figs. 4, 6 and 7). These vapor columns and vapor patches are attached to the heating surface by numerous stems (cf. Figs. 4 and 6).

(11) In this regime the vapor is produced (most probably) by continuous vaporization of a pulsating micro-layer (described and proposed by Moore and Mesler) at the base of vapor columns or of vapor patches. The vapor is then transmitted through the columns to form large vapor slugs (cf. Figs. 4, 6 and 7).

(12) In this regime the dominant heat-transfer mechanism is, most probably, the latent heat transport process formulated by Gaertner.

(13) The limit of the region of interference is

given by the heat flux corresponding to the upper limit of the region of isolated bubbles (58) and by the critical heat flux (59). (Cf. Fig. 13).

ACKNOWLEDGEMENT

The author is indebted to Mr. R. H. Norris, from the General Engineering Laboratory, and to Dr. R. F. Gaertner, from the Research Laboratory, for a critical review of the paper and for many stimulating discussions.

The author is indebted also to Mr. R. Semmeria from the Commissariat à l'Énergie Atomique, Centre d'Études Nucleaires de Grenoble, France, and to Dr. R. F. Gaertner from the General Electric Research Laboratory, Schenectady, New York, for giving him the photographs of the boiling process shown in this paper.

REFERENCES

1. N. Zuber and E. Fried, Two-phase flow and boiling heat transfer to cryogenic liquids, *J. Amer. Rocket Soc.* **32**, 1332 (1962).
2. P. Sachs and R. A. K. Long, A correlation for heat transfer in stratified two-phase flow with vaporization, *Int. J. Heat Mass Transfer*, **2**, 222 (1961).
3. W. R. Rohsenow and H. Y. Choi, *Heat, Mass and Momentum Transfer*, pp. 210–236. Prentice–Hall, New York (1961).
4. J. W. Westwater, Things we don't know about boiling heat transfer. Paper presented at the 1960 Annual ASME Meeting, New York.
5. C. Courty and A. Foust, Surface variables in nucleate boiling, *Chem. Engng Progr.*, Symp. Ser. No. 17, **51**, 1 (1955).
6. P. Griffith and J. D. Wallis, The role of surface conditions in nucleate boiling, *Chem. Engng Progr.*, Symp. Ser. No. 30, **56**, 49 (1960).
7. N. Zuber and M. Tribus, Further remarks on the stability of boiling heat transfer. *AEC* Rep. *AECU* 3631, *AEC* Tech. Inf. Serv., Oak Ridge, Tenn. (1958).
8. N. Zuber, Hydrodynamic aspects of nucleate pool boiling, Part I—The region of isolated bubbles. Rep. RW-RL-164, January (1960), *AEC* Rep. T.I.D. 6338. Office Tech. Ser., Dept. of Com., Wash., D.C.
9. P. J. Berenson, Experiments in pool-boiling heat transfer, *Int. J. Heat Mass Transfer*, **5**, 985 (1962).
10. B. Stock, Observation on transitional boiling. Argonne Nat. Lab. Rep. *ANL* 6175 (1960).
11. K. Yamagata, F. Hirano, K. Nishikawa and H. Matsuoka, Nucleate boiling of water on a horizontal surface, *Mem. Fac. Engr. Kyushu Univ.* **15**, 1, 97 (1955).
12. K. Nishikawa and K. Yamagata, On the correlation of nucleate boiling heat transfer, *Int. J. Heat Mass Transfer*, **1**, 219 (1960).
13. H. M. Kurihara and J. E. Myers, Fundamental factors affecting boiling. *Amer. Inst. Chem. Engrs J.* **6**, 83 (1960).
14. R. F. Gaertner and J. W. Westwater, Population of active sites in nucleate boiling heat transfer, *Chem. Engng Progr.*, Symp. Ser. No. 30, **46**, 39 (1960).
15. C. L. Tien, A hydrodynamic model for nucleate pool boiling, *Int. J. Heat Mass Transfer*, **5**, 533 (1962).
16. M. Jakob and W. Linke, Der Warmeübergang beim Verdampfen von Flüssigkeiten an Senkrechten und Waagerechten Flächen, *Phys. Z.* **36**, 267 (1935).
17. M. Jakob and W. Fritz, Versuche über den Verdampfungsvorgang. *Forschung Ver. Dtsch Ing.* **2**, 12, 435 (1931).
18. W. R. Malkus, Discrete transitions in turbulent convection, *Proc. Roy. Soc.* A **225**, 185 (1954).
19. W. R. Malkus, The heat transport and spectrum of thermal turbulence, *Proc. Roy. Soc.* A **225**, 196 (1954).
20. D. B. Thomas and A. A. Townsend, Turbulent convection over a heated horizontal surface, *J. Fluid Mech.* **2**, 473 (1957).
21. A. A. Townsend, Temperature fluctuations over a horizontal heated surface, *J. Fluid Mech.* **5**, 209 (1959).
22. M. Jakob, Kondensation und Verdampfung. *Z. Ver. Deutsch. Ing.* **76**, 1161 (1932).
23. M. Jakob, *Heat Transfer*, Vol. 1, Ch. 29, Wiley, New York (1949).
24. W. Rohsenow, A method of correlating heat transfer data for surface boiling of liquids. *Trans. ASME*, **74**, 969 (1952).
25. W. Rohsenow and J. Clark, A study of the mechanism of boiling heat transfer. *Trans. ASME*, **73**, 609 (1951).
26. F. C. Gunther and F. Kreith, Photographic study of bubble formation in heat transfer to sub-cooled liquids. Heat transfer and Fluid Mechanics Institute.
27. H. K. Forster and N. Zuber, Bubble dynamics and boiling heat transfer, *Amer. Inst. Chem. Engrs J.* **1**, 532 (1955).
28. S. S. Kutateladze, *Heat Transfer in Condensation and Boiling*. 2nd Ed. Mashgiz, Moscow (1952). *AEC* Translation 3770, U.S. *AEC* Tech. Inf. Service, Oak Ridge, Tenn.
29. G. N. Kruzhilin, Heat transmission from a heating surface to a boiling one-component liquid in free convection. *Izv. Akad. Nauk SSSR, Otdel. Tekh. Nauk.* p. 967 (1948), *AEC* Trans. RT 2792.
30. G. N. Kruzhilin, Generalization of experimental data on heat transfer to boiling liquids in free convection, *Izv. Akad. Nauk SSSR, Otdel. Tekh. Nauk.* No. 3, p. 701 (1949), *AEC* Trans. RT 2542.
31. F. D. Moore and R. B. Mesler, Micro-layer vaporization—a new hypothesis about nucleate boiling based on recent experimental evidence, *Amer. Inst. Chem. Engrs J.* **7**, 620 (1961).
32. S. G. Bankoff and J. P. Mason, Heat transfer from the surface of a steam bubble in a turbulent subcooled liquid, *Amer. Inst. Chem. Engrs J.* **8**, 30 (1962).
33. Y. Y. Hsu and R. Graham, An analytical and experimental study of the thermal boundary layer and ebullition cycle in nucleate boiling. *N.A.S.A.* Rep. TN-D-594 (1961).
34. S. S. Kutateladze, A hydrodynamic theory of changes in a boiling process under free convection. *Izv. Akad. Nauk SSSR, Otdel. Tekh. Nauk.* No. 4, 529 (1951).

35. N. Zuber, On the stability of boiling heat transfer, *Trans. ASME*, **80** 711 (1958).
36. N. Zuber, Hydrodynamic aspects of boiling heat transfer. U.S. *AEC* Rep. *AECU* 4439 Tech. Inf. Serv. Oak Ridge, Tenn. (1959).
37. N. Zuber, M. Tribus and J. W. Westwater, The hydrodynamic crisis in pool boiling of saturated and sub-cooled liquids. Paper No. 27, 1961 Int. Heat Transfer Conf., Boulder, Colorado (1961).
38. W. Fritz and W. Ende, Verdampfungsvorgang nach Kinematographischen Aufnahmen und Dampfblasen, *Phys. Z.* **37**, 391 (1936).
39. B. E. Staniszewski, Nucleate boiling bubble growth and departure. Tech. Report No. 16, Heat Transfer. Lab. Mass. Inst. of Tech., Cambridge, Mass., Arg. (1959).
40. P. H. Strenge, A. Orell and J. W. Westwater, Microscopic study of bubble growth during nucleate boiling, *Amer. Inst. Chem Engrs J.* **7**, 578 (1961).
41. S. G. Bankoff, On the mechanism of sub-cooled nucleate boiling—I. Preliminary considerations, *Chem. Engng Progr.*, Symp. Ser. No. 32, **57**, 156 (1960).
42. S. G. Bankoff, On the mechanism of sub-cooled nucleate pool boiling—II. Sequential rate process model, *Chem. Engng Progr.*, Symp. Ser. No. 32, **57**, 164 (1960).
43. W. Fritz, Berechnung des Maximalvolumes von Dampfblasen, *Phys. Z.* **36**, 379 (1935).
44. F. N. Peebles and H. J. Garber, Studies on the motion of gas bubbles in liquids, *Chem. Engng Progr.* **49**, 88 (1953).
45. T. Harmathy, Velocity of large drops and bubbles in media of infinite or restricted extent. *Amer. Inst. Chem. Engrs J.* **6**, 281 (1961).
46. P. Grassmann, *Physikalische Grundlagen der Chemie Ingenieur—Technik*, p. 752. Verlag Sauerlander, Frankfurt am Main (1961).
47. J. W. Westwater and J. G. Santangelo, Photographic study of boiling, *Industr. Engng Chem.* **47**, 1605 (1955).
48. J. W. Westwater and A. S. Perkins, Measurement of bubbles formed in boiling methanol. *Amer. Inst. Chem. Engrs J.* **2**, 471 (1956).
49. L. Davidson and E. Amick, Formation of gas bubbles at horizontal orifices. *Amer. Inst. Chem. Engrs J.* **2**, 336 (1956).
50. D. W. van Krevelen and P. J. Hoftijzer, Studies of gas bubble formation, *Chem. Engng Progr.* **46**, 29 (1950).
51. W. Siemes, Über die Erzeugung und den Aufsteig von Gasblasen in Flüssigkeiten, *Chem.-Ing.-Tech.* **26**, 479 (1954).
52. W. Siemes, Gasblasen in Flüssigkeiten, *Chem.-Ing.-Tech.* **26**, 614 (1954).
53. I. Leibson, E. G. Holcomb, A. C. Cacoso and T. Tacmic, Rate of flow and mechanics of bubble formation from single submerged orifices. *Amer. Inst. Chem. Engrs J.* **2**, 296 (1956).
54. T. F. Davidson and B. O. Schuler, Bubble formation in an orifice in an inviscid liquid, *Trans. Instn. Chem. Engrs, Lond.* **38**, 335 (1960).
55. J. W. Westwater, Boiling heat transfer, *Amer. Scient.* **47**, 427 (1959).
56. R. Cole, A photographic study of pool boiling in the region of the critical heat flux. *Amer. Inst. Chem. Engrs J.* **6**, 533 (1960).
57. G. B. Wallis, A gas-liquid analogue of nucleate boiling, *Nuclear Power* **5**, 100 (1960).
58. R. F. Gaertner, The distribution of active sites in the nucleate boiling of liquids. General Electric Research Laboratory Report, No. 61-RL-2826C, September (1961), Schenectady, N.Y., *Amer. Inst. Chem. Engr.* No. 5, *AIChE–ASME* Heat Transfer Conference, Houston, Texas (1962).
59. G. B. Wallis, Some hydrodynamic aspects of two-phase flow and boiling. Paper No. 38, 1961 International Heat Transfer Conference, Boulder, Col. (1961).
60. H. Schulman and M. S. Molstad, Gas bubble columns for gas–liquid contacting, *Industr. Engng Chem.* **42**, 1059 (1950).
61. H. Kolbel and W, Siemes, Eigenshaften von Blasensaulen, *Dechema-Monograph* **29**, 334 (1957).
62. G. Houghton, A. M. McLean and P. D. Ritchie. Mechanism of formation of gas-bubble-beds, *Chem. Engng Sci.* **7**, 40 (1957).
63. N. Zuber and J. Hench, Steady state and transient void fraction of bubbling systems and their operating limits. Rep. No. 62GL100, General Engineering Laboratory, General Electric Co., Schenectady (1962).
64. R. F. Gaertner, Boiling heat transfer—I. A photographic study of nucleate boiling. General Electric Co. Research Laboratory Report, Schenectady, N.Y. (1962) (To be published).
65. R. F. Gaertner, Boiling heat transfer—II. The mechanism of high-flux nucleate boiling. General Electric Co. Research Laboratory Report, Schenectady (1962) (To be published).
66. J. M. Coulson and J. F. Richardson, *Chemical Engineering*, Vol. II. McGraw-Hill, New York (1955).
67. S. T. Hsu and F. W. Schmidt, Measured variations in local surface temperatures in pool boiling of water, *Trans. ASME J. Heat Transfer*, **83**, 254 (1961).
68. R. Siegel and C. Usiskin, A photographic study of boiling in the absence of gravity, *Trans. ASME J. Heat Transfer* **81**, 230 (1959).
69. H. Merte and J. A. Clark, Pool boiling in an accelerating system, *Trans. ASME J. Heat Transfer* **83**, 233 (1961).
70. R. Moisis and P. J. Berenson, On the hydrodynamic transitions in nucleate boiling. *ASME* paper 62-HT-8, *ASME-AIChE* Heat Transfer Conference, Houston, Texas (1962).
71. F. O. Mixon, W. Y. Chon and K. O. Beatty, The effect of electrolytic gas evolution on heat transfer, *Chem. Engng Progr.*, Symp. Ser. No. 30, **56**, 75 (1960).
72. G. B. Wallis, Two-phase flow aspects of pool boiling from a horizontal surface. Atomic Energy Authority, England, Rep. AEEW-R-103 (1961).

APPENDIX A

A mean liquid velocity due to the source flow

It was assumed by Bankoff [41, 42] that nucleate sub-cooled boiling can be approximated by a system of sources and sinks the velocity in the liquid due to one source being given by

$$u(r) = \frac{R^2 \mathring{R}}{r^2} \tag{A-1}$$

where R and $\mathring{R}$ are the bubble radius and radial velocity. It was shown in [8] that (A-1) can be used to estimate a mean liquid velocity due to the source flow of the liquid within a bubble "influence domain" defined by

$$s^2 = \frac{A}{n}. \tag{A-2}$$

defining the mean velocity by

$$\bar{U}(r) = \frac{1}{(s/2) - R} \int_R^{s/2} \frac{R^2 \mathring{R}}{r^2} \, \mathrm{d}r \tag{A-3}$$

then

$$\bar{U}(r) = \frac{2R\mathring{R}}{s}. \tag{A-4}$$

It follows from (A-4), (A-2) and (4) that for liquids at saturation

$$\bar{U}(r) = 2R\mathring{R} \sqrt{\left(\frac{n}{A}\right)}$$
$$= \frac{4}{\pi^2} \left[\frac{(T_w - T_s)\, c\rho_L \sqrt{(\pi a)}}{\rho_v h_{fg}}\right]^2 \sqrt{\left(\frac{n}{A}\right)}. \tag{A-5}$$

As anticipated, the velocity of the liquid caused by the radial flow depends upon both the rate of bubble growth and the bubble population density.

It is interesting to note that a similar expression can be obtained for sub-cooled liquids if one uses the product $2R\mathring{R}$ derived by the author in the paper entitled: "The dynamics of vapor bubbles in non-uniform temperature fields", *Int. J. Heat Mass Transfer*, **2**, 82 (1960).

APPENDIX B

The product $D_b . f$

Peebles and Garber [44] found from experiments performed with a large variety of liquids that deformed bubbles rise with a velocity given by (6). Jakob [22, 23], (Fig. 29-14) reported that the velocity or rise of a bubble immediately after departure was 17 cm/s. For water at atmospheric pressure and at saturation temperature (6) predicts a velocity of rise $U_T = 18{\cdot}6$ cm/s. The application of (6) to pool boiling appears therefore permissible.

Let x be the center-to-center spacing between two consecutive bubbles, for a constant rise velocity then

$$x = (t_d + t_b)U_t \tag{B-1}$$

where t_d and t_b are the delay time and the bubble break-off time (time at departure) respectively. Define

$$x = mD_b \tag{B-2}$$

where m is a number, i.e. amultiplier. Then from (B-1) and (B-2) it follows that

$$\frac{mD_b}{t_b\,(1 + t_d/t_b)} = U_t. \tag{B-3}$$

When the delay time $t_d \ll t_b$, and at low pressures when bubbles grow rapidly and follow each other closely, then $m \approx 1$. Consequently from (6) and (B-3) it follows that

$$\frac{D_b}{t_b} \approx D_b . f = 1{\cdot}18 \left[\frac{\sigma g(\rho_L - \rho_v)}{\rho_L{}^2}\right]^{1/4}. \tag{B-4}$$

Jakob and Linke [16] found that while the bubble still adheres to the surface its center of gravity rises with almost the same velocity with which the bubble later rises when it leaves the surface (see also [23] p. 631). For their experimental conditions Jakob and Linke found also that the delay time t_d, was almost equal to the bubble break-off time, t_b. These two observations imply that the following relations are approximately valid

$$\frac{D_b}{t_b} \approx U_t \tag{B-5}$$

and

$$f = \frac{1}{2t_b}. \tag{B-6}$$

From (B-5), (B-26) and (24) then:

$$D_b . f = \frac{1{\cdot}18}{2} \left[\frac{\sigma g\,(\rho_L - \rho_v)}{\rho_L{}^2}\right]^{1/4} \tag{B-7}$$

which is equation (9) in the text. Equation (B-3) implies that, for the conditions given by (B-5) and (B-6), m is equal to 2, i.e. that the center to center spacing between a departing bubble and its predecessor is $x = 2D_b$. This result is in approximate agreement with the spacing shown on Fig. 29-12 in reference [23].

APPENDIX C

Some comments by Townsend on turbulent natural convection from horizontal surfaces

The most striking feature revealed by the experiments of Thomas and Townsend [20] and Townsend [21] and analysed by them with great insight, was that the fluctuations of temperature, of temperature gradients, and rate-of-change of temperature, all exhibited periods of activity, characterized by large fluctuations, alternating with periods of quiescence with comparable small fluctuations. Townsend remarked; "If the temperature at a fixed point has two distinct modes of fluctuations, the space occupied by the fluid at any instant must be divided by comparatively well-defined bounding surfaces into corresponding regions of 'activity' and 'quiescence', and the properties of the modes are also those of the regions. Within the active regions temperature fluctuations are large and the mean temperature is high, while within the quiescent regions the fluctuations are much smaller and the mean temperature only slightly above the reference temperature".

The observed fluctuations were statistically homogeneous over horizontal planes. Both the proportion and the frequency of occurrence of active periods decreased with increasing distance from the surface and probably occurred when rising columns of hot air passed through the point of observation. Townsend concluded that the quiescent fluctuations are typical of the turbulent convection far from the surface while the active fluctuations are a manifestation of the convective processes arising near the rigid boundary. These processes were described as the detachment of columns of hot air from the edge of the conduction layer and the erosion of these rising columns by contact with the surrounding air which is in vigorous turbulent motion.

In describing the flow adjacent to the surface, Townsend observed further: "It appears probable that active regions are formed by more or less localized emission of heat from the conduction layer, most likely in the neighborhood of points or lines of flow 'separation' where the horizontal velocity just outside the conduction layer happens to be small or zero". . . . "The presence and persistence of 'up-draught sites' has been described by Malkus who observed the motion of suspended particles in acetone and water, and these may be identified with the hypothetical heat sources. The persistence of the sources is confirmed in these experiments by the comparatively long duration of the active periods (of the order of 10 s) compared with a scale time of about 0·5 s, and a possible explanation is that, once a site is established, it attracts to itself air heated by passage through the conduction layer which adds to the strength and stability of the up-draught".

According to Townsend "the distribution of mean temperature is determined by the extent to which these up-draughts penetrate the cool 'quiescent' air, which is dependent on the initial cross-section and strength of the up-draught, both closely related to the thickness of the conduction layer."

Résumé—Les données expérimentales indiquent que l'ébullition nucléée comprend deux régions: (a) la région des bulles isolées et (b) la région d'interférence. Les modèles du transport de vapeur, d'écoulement et les mécanismes de transport de chaleur dans les deux régions sont discutés et étudiés. Un critère pour le passage d'une zone à l'autre est présenté.

Dans le régime des bulles isolées, il n'y a pas interférence entre les bulles et la vapeur se produit de façon intermittente en des points particuliers. La description de Jakob de la convection naturelle dans un liquide en ébullition nucléée au-dessus d'une surface horizontale est semblable à celle de Malkus et Townsend pour le régime d'écoulement en convection libre turbulente au-dessus d'un plan horizontal. Dans les deux cas, le transport de chaleur est provoqué par les courants convectifs. On montre que des équations semblables à celles qui donnent le coefficient de transmission de chaleur et la variation de vitesse moyenne turbulente dans le cas de la convection libre turbulente au-dessus d'un plan horizontal peuvent être utilisées dans le cas des bulles isolées si le coefficient d'évacuation de vapeur, c'est-à-dire la vapeur enlevée est prise en considération dans l'évaluation de la densité moyenne du fluide. On donne les équations reliant le coefficient d'évacuation de la vapeur au coefficient de transport de chaleur ou à la densité de population en bulles et à la température du liquide surchauffé. On montre qu'il existe une limite supérieure au mécanisme de transmission de chaleur induit par les courants. Les équations donnant la valeur limite du coefficient de transmission de chaleur et de la densité du flux thermique dans le régime des bulles isolées sont également présentées. Tous ces résultats analytiques se révèlent qualitativement et quantitativement en bon accord avec les données expérimentales actuelles.

Dans la zone d'interférence, les bulles se collent les unes aux autres pour former des colonnes continues et des masses de vapeur. La vapeur est produite en permanence par vaporisation d'une micro couche "oscillante" (proposée et décrite par Moore et Mesler) à la base d'une colonne ou d'une masse de vapeur. Dans ce régime, le mécanisme dominant de transmission de chaleur est, le plus probablement, le processus de transport de chaleur latente (formulé par Gaertner) et le transport de chaleur latente associé aux gros éclatements de vapeur provoqués par l'affaissement des masses de vapeur.

Les résultats de l'étude indiquent qu'un régime particulier d'ébullition nucléée (problème à deux phases) peut être étudié comme un problème de convection turbulente naturelle (problème à une seule phase). Les applications de considérations semblables à d'autres formes d'écoulement à deux phases apparaissent alors pleine de promesses.

L'important effet des modèles d'écoulement à deux phases sur le mécanisme de transmission de chaleur et sur le coefficient d'échange d'un mélange à deux phases est encore démontré et étudié.

Zusammenfassung—Nach Versuchsergebnissen erfolgt die Blasenverdampfung in zwei Bereichen: (a) dem Bereich der Einzelblasen und (b) dem Bereich gegenseitiger Blasenbeeinflussung. Die Dampfabfuhr, die Strömungsart und der Wärmetransportmechanismus in beiden Bereichen werden diskutiert und analysiert. Ein Kriterium für den Übergang von einem Bereich in den anderen ist angegeben. Im Bereich der Einzelblasen entsteht Dampf intermittierend an den Keimstellen und die Blasen stören sich gegenseitig nicht. Jakobs Beschreibung der Zirkulationsströmung beim Blasensieden unter freier Konvektion an einer waagerechten Fläche ist ähnlich der Beschreibung von Malkus und Townsend, des Strömungsverlaufs bei turbulenter freier Konvektion über einer waagerechten Oberfläche. In beiden Fällen beruht der Wärmetransport auf der vom Auftrieb hervorgerufenen Zirkulation. Es zeigt sich, dass die Gleichungen, die den Wärmeübergangskoeffizienten und die mittlere turbulente Geschwindigkeitsschwanking der turbulenten freien Konvektion über einer waagerechten Oberfläche bestimmen, auch für den Bereich der Einzelblasen anwendbar sind, wenn zur Berechnung der mittleren Flüssigkeitsdichte der Dampfraumkoeffizient d.h. der Dampfanteil berücksichtigt wird. Gleichungen, die den Dampfraumkoeffizienten mit dem Wärmeübergangskoeffizienten oder mit der Blasendichte und der Überhitzungstemperatur der Flüssigkeit verbinden, sind angegeben. Der Wärmetransportmechanismus, wie er von der Auftriebszirkulation hervorgerufen wird, besitzt eine obere Grenze. Gleichungen für den Grenzwert des Wärmeübergangskoeffizienten und der Wärmeflussdichte im Bereich der Einzelblasen sind angegeben. Alle diese, durch die Analyse erhaltenen Ergebnisse stimmen mit gegenwärtig verfügbaren Versuchsresultaten qualitativ und quantitativ gut überein.

Im Bereich gegenseitiger Blasenbeeinflussung stören sich die Blasen als kontinuierliche Dampfsäulen oder Ballen. Durch Verdampfung einer pulsierenden Mikroschicht (wie sie von Moore und Mesler vorgeschlagen und beschrieben wurde) am Fuss jeder Dampfsäule und jedes Dampfballens wird fortwährend Dampf nachgeliefert. In diesem Bereich beruht der Mechanismus des Hauptwärmetransports höchstwahrscheinlich auf dem Transport latenter Wärme (wie von Gärtner formuliert) und dem Transport latenter Wärme in Verbindung mit der ausgedehnten Dampfverteilung beim Zusammenbruch der Dampfballen.

Die Ergebnisse der Untersuchung zeigen, dass ein Teilbereich des Blasensiedens unter freier Konvektion (ein Zweiphasenproblem) als ein Problem der turbulenten freien Konvektion (als ein Einphasenproblem) analysiert werden kann. Die Anwendung ähnlicher Betrachtungen auf andere Gegebenheiten der Zweiphasenströmung scheinen somit erfolgversprechend. Der wichtige Einfluss der Art der Zweiphasenströmung auf den Mechanismus des Wärmetransports und auf den Wärmeübergangskoeffizienten eines Zweiphasengemisches ist wiederum gezeigt und besonders diskutiert.

Аннотация—Экспериментальные данные показывают, что пузырьковое кипение состоит из двух областей, а (область изолированных пузырьков и б) область пленочного кипения. Рассматриваются и анализируются картина отвода пара, картина потока и механизмы теплообмена в двух областях. Дается критерий перехода от одной одласти к другой.

В условиях изолированных пузырьков пузырьки не влияют друг на друга и в любой точке пар вырабатывается отдельными порциями. Описание Якоба для естественной циркуляции потока при пузырьковом кипении с горизонтальной поверхности аналогично описанию Малкуса и Таунсенда потока для режима при турбулентной естественной конвекции с горизонтальной поверхности. В обоих случаях теплообмен вызван вынужденной циркуляцией восходящего течения. Показано, что те уравнения, которые определяют коэффициент теплообмена и флуктуацию средней турбулентной скорости при естественной турбулентной конвекции с горизонтальной поверхности, могут быть использованы в области изолированных пузырьков, если коэффициент содержания паровой фракции, т.е. учитывается при определении средней плотности потока. Приводятся уравнения, которые устанавливают связь между коэффициентом содержания паровой фракции и коэффициентом теплообмена или между плотностью пузырьков и температурой перегретой жидкости. Показано, что существует верхний предел для механизма теплообмена, вызванного циркуляцией обусловленной естественной конвекцией. Также даются уравнения, определяющие предельное значение коэффициента теплообмена и плотности теплового потока в условиях изолированных пузырьков. Показано, что все эти результаты, определенные с помощью анализа, согласуются качественно и количественно с экспериментальными данными, имеющимися в настоящее время.

В области пленочного кипения пузырьки взаимодействуют, образуя участки сплошных потоков пара. Пар непрерывно создается испарением пульсирующего микрослоя (предложено и описано Моором и Меслером). При этом условии, весьма возможно, что преобладающим является в механизме теплообмена процесс переноса скрытой теплоты (сформулировано Хартнером), а перенос скрытой теплоты, связан с большим числом взрывов пузырьков пара, в связи с разрушением паровых участков.

Результаты анализа показывают, что частный режим пузырькового кипения (двухфазная задача) может быть проанализирован как задача турбулентной естественной конвекции (однофазная задача). Поэтому применение такого анализа с другим аспектам двухфазного потока представляется очень перспективной. В заключении снова рассматривается влияние структуры двухфазного потока на механизм теплообмена и на коэффициент теплообмена для двухфазной смеси.